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STATISTICAL STUDY OF PRIMER SENSITIVITY DROP-TESTS

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WHITE OAK, MARYLAND

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STATISTICAL STUDY OF PRIMER SENSITIVITY DROP-TESTS

INTRODUCTION

1. Sensitivity is an important characteristic of initiating elements and one which is frequently determined during the development of primers and detonators. During the development work where sample size is limited and where high accuracy of results is not essential, sensitivity is determined by the up-and-down (Bruceton) technique. Interval size is chosen arbitrarily as dictated by the experience of the operator.

2. In the sensitivity testing of mechanically initiated components, arithmetic intervals are used. In those cases where the sensitivity pattern does not follow a normal distribution, it occasionally happens that calculations of low percentage firing heights give negative (impossible) values. In the sensitivity testing of electric initiators logarithmically spaced intervals have been used extensively. One advantage of this is the elimination of predicted negative firing heights.

3. Where accuracy of a higher degree than that necessary during the development work is required, such as in preliminary design and design proof tests, the rundown technique is used. Here again the experience of the operator dictates the interval size.

4. The arithmetic interval has long been used in mechanical primer and detonator testing, although there is no known statistical evidence of its superiority over the logarithmic interval. It was decided, therefore, that a study of sensitivity testing be made in order to increase the quantity and quality of information which might be obtained from the samples available for test.

5. Two types of primers were used in this study, percussion type and stab type. The percussion primer was the Mk 101 type containing an experimental priming mixture (NOL No. 17) of the following composition:

Basic lead styphnate	20%
Barium nitrate	35%
Tetracene	5%
Antimony sulfide	20%
RDX	20%

Two stab type primers were used in the study -- the Mk 102 Mod 0 and a Mk 120 type primer. Since the two types of primers exhibited somewhat different sensitivities, each type will be discussed individually.

The Mk 101 Type Primer (Percussion)

Test Procedure

6. A group of Mk 101 primers containing #17 explosive mix and supposedly manufactured under constant conditions was employed throughout the study. A series of four tests of the Bruceton or "up and down" type was first conducted. The Bruceton test, now used extensively at NOL, is fully described in reference 1. Each test consisted of 200 trials. Two of the tests were conducted at drop heights equally spaced at 1/4 inch and 1/2 inch intervals, respectively, on the arithmetic scale. The remaining two tests were run at drop heights approximately equally spaced at .05 and .10 intervals, respectively, on the logarithmic scale. In this way it was hoped that information could be obtained relative to the effect of: (a) arithmetic vs. logarithmic intervals, and (b) variation in interval size, on the accuracy of the results obtained. Furthermore, by breaking the 200 trial Bruceton tests down into smaller samples, it was hoped that information could be obtained on the effect of sample size on the precision of estimates obtained by the Bruceton method.

7. Following the running of the Bruceton tests described above, enough additional trials were conducted so as to obtain a complete rundown with a minimum of 100 trials at 1/4 inch intervals. The great majority of the trials already run in the Bruceton tests were incorporated into the rundown. The Bruceton tests thus comprised sub-samples of the large rundown sample of 1439 trials. Throughout the trials the selection of the primers was randomized so as to minimize the effect of a possible sequence of non-representative primers.

8. The data from the complete rundown were analyzed by the method of probits to be described below. This analysis produced estimates of the 1%, 50% and 99% firing points which were used as norms against which the Bruceton estimates could be compared. The probit analysis also produced tests of the "goodness-of-fit" of the experimental data to the theoretical normal distribution.

9. The validity of the Bruceton method of obtaining estimates is based on the assumption that the explosion percentages follow a cumulative normal distribution when plotted against height or whatever function of height is employed in equally spacing the intervals at which trials are conducted. Hence the comparison of the "goodness of fit" of the data as analyzed on both the arithmetic and

logarithmic scales was perhaps the most important aspect of the probit analysis.

10. The data of the various Bruceton tests and subtests was then analyzed using the method described in reference 1. Estimates of the various percentage points were obtained and the assumption of normality was further tested.

11. The statistical technique known as Probit Analysis is fully discussed in reference 2, in connection with the problems of biological assay. However, it is readily seen to be adaptable to the analysis of the results of sensitivity tests on explosives and in this connection it is further discussed in references 3 and 6. A brief description of the method follows.

12. A number of trials are first conducted at various heights which need not be equally spaced. The number of explosions at each height is recorded and the corresponding percentage is computed. These experimentally obtained percentages are then transformed to quantities called probits by consulting a table. The probits are then plotted against height or some function of the height on ordinary graph paper. These probits are ordinates of a certain normal distribution and are such that the points plotted on the graph referred to above would lie in a straight line if the explosion percentages were truly normally distributed. If the conditions of normality are reasonably well approximated by the experimental data the plotted points will exhibit a marked linear trend.

13. The next portion of the probit analysis consists of fitting a line, known as a regression line, to the points plotted on the graph. The method employed is an iterative one. A line which seems to best fit the data is first drawn by observation. If the points cluster very closely about this line, no further refinement may be necessary and the line thus drawn may be employed in the remainder of the analysis. However, if this is not the case, a better-fitting line can be obtained by an arithmetical procedure which involves weighting factors which depend upon both the number of trials conducted at each level and the distance of the level from the 50% point of the distribution. In determining the fitted line a point near the 50% point carries considerably more weight than one near an extreme. The weighting factors also increase with the number of trials. This arithmetical procedure can be carried through any number of cycles until what is for practical purposes the "best-fitting" line is obtained.

14. The criterion of best fit in the arithmetical procedure described above is the convergence, in the practical

sense, of two successive cycles of computation. In other words, when a line is obtained which is, for practical purposes, identical with the line obtained from the preceding cycle of computation, then the best fit is adjudged to have been attained.

15. The fact that a regression line has been obtained which is for practical purposes the best-fitting line for the data does not, however, necessarily imply that it is a good fit. The "goodness of fit" of the best-fitting line (or of any of the preceding lines) can be measured by the well-known χ^2(chi-square) test. The test is thus a criterion of the validity of the assumption of normality in the distribution of the firing percentages. If by this standard the best fitting line shows a reasonably good fit, then its equation may be used to estimate the point at which any given percentage of the explosive units will fire. Confidence intervals for these estimates can also be computed. If, by the χ^2 criterion, it is impossible to obtain a reasonably good fit for the plotted points, then these confidence intervals must be increased by the application of a heterogeneity factor. In other words, if the normality assumption seems improbable, then a wider range of possible variation must be allowed for in the estimates obtained from the probit regression equation.

Results

16. The results of the complete rundown are shown in Cols. 1, 2 and 3 of Data Sheet 1. All of the trials conducted in the Bruceton tests were incorporated into this rundown except for a few levels at which only a very small number of trials were conducted. These could have only a negligible effect on the probit line, while at the same time they would increase the labor involved in the arithmetical process of fitting the line.

17. The calculations involved in the probit analysis of the data are shown on Data Sheets 1, 2 and 3, and the graphical picture is presented by Figures 1 and 2. The probits corresponding to the empirical explosion percentages are shown in column 5 of Data Sheet 1 and these probits are plotted against height in Figure 1 and against log height in Figure 2. The solid line on each of these graphs is the line drawn by observation referred to in the preceding description of the probit analysis. The various cycles of arithmetical calculations involved in obtaining the best-fitting regression line are shown. The first cycle of computations involved in the fitting of a line to the empirical probits plotted in Figure 1 are shown in columns 6 through 10 of Data Sheet 1. The corresponding calculations for Figure 2 are shown in columns 11 through 16

of Data Sheet 1. The succeeding cycles of calculation are shown on Data Sheets 2 and 3. These calculations are described in detail in reference 2, pages 199-208.

18. The final adjusted lines are the dashed lines shown in Figures 1 and 2. The goodness of fit of the adjusted lines at each stage of the calculations is indicated by the following values, which can be readily obtained from the computations involved in the fitting of the line with only slight additional labor.

	Height	Log Height
1st Cycle	8.232	10.410
2nd Cycle	8.065	8.811
3rd Cycle	8.405	8.787

The subscript 11 on each of these χ^2's indicates that it has 11 degrees of freedom. The average value of such a χ^2 function is 11. Hence any value less than 11 indicates a reasonably good fit and supports the hypothesis of normality. Conversely, a value considerably in excess of 11 would cast doubt on the normality hypothesis.

19. By this criterion the fit is a good one for both height and log height, with the former enjoying a very slight advantage. It will be noted that in the case of height the χ^2 value for the final line is slightly higher than for the preceding lines. This is due to the fact, alluded to in reference 2, page 54, that the process of obtaining the best-fitting line in the sense of convergence is not precisely equivalent to minimizing the χ^2 value though the final χ^2 value will usually not differ by much from the minimum.

20. These figures would tend to the conclusion that good results can be obtained in Bruceton tests run at both arithmetically and logarithmically spaced intervals. The superiority enjoyed by the former seems too slight to enforce the conclusion that it is definitely better. Hence factors other than theoretical normality will influence the choice of intervals, and this matter will be discussed in a later portion of this report.

21. Attention will next be focused on the equation of the regression line which will be used to estimate the various percentage points. By the χ^2 criterion the line which best represents the experimental data is that which expresses the functional relationship between probit and height and whose equation is:

1. $Y = -.532 + 2.182X$

or 2. $X = \dfrac{Y + .532}{2.182}$

If the probit corresponding to any given percentage is substituted for Y in equation 2, then the Height X corresponding to that percentage can be readily obtained. In this way the following estimates were obtained:

1% Height:	1.47 inches
50% Height:	2.54 inches
99% Height:	3.60 inches

22. These would appear to be the best estimates of the given percentage points. However, since log height also appears to be a reasonably good normalizing function it seems pertinent to record the corresponding percentage heights as estimated from the best regression equation connecting probit with log height. This equation is:

$$X^1 = \frac{Y + .096}{12.782}$$

23. The various percentage heights are obtained in the same way as previously except that X^1 is now a logarithm and must be transformed back to the original units of height. In this way the following estimates are obtained.

1% Height:	1.65 inches
50% Height:	2.51 inches
99% Height:	3.81 inches

It will be noted that the estimates of the 50% height as obtained by the arithmetic and logarithmic scales differ only slightly, whereas in the case of estimates of the extreme points the agreement is not so good.

24. A question may arise as to the reason for as large a discrepancy as was shown in the estimation of the extreme point by the two scales when both were adjudged by the χ^2 criterion to give reasonably good fits to the normal regression line.

25. The answer to this question lies partly in the fact that the χ^2 criterion expresses the <u>average</u> goodness of fit over the entire regression line, with values at the extreme points of the line contributing but little to the computed χ^2 value. Therefore, the major, central portion, of both regression lines may fit the experimental data well and the χ^2 values may be comparable, but the extremes of at least one of the lines must deviate since both assumptions of normality, i.e., with height or log height, cannot be correct. It should be noted also that the nature of the logarithmic function is such that it will always give higher estimates of the extreme points (both the 1% and 99% points) no matter which assumption of normality is correct. For when the sensitivity is normal with regard to height the logarithmic plot will have its extremes bend

upward away from the regression line, and when the sensitivity is normal with respect to log height, the experimental values of the extremes plotted on the height basis will bend downward away from the regression line. In either case the extremes (both high and low) will be higher on the log height basis.

26. The statistics given above are estimates of the precise points at which given percentages of the primers under consideration may be expected to fire. However, like all statistical estimates of this kind, they may be expected to vary from sample to sample. Hence they should be surrounded by certain ranges of variation known as confidence intervals. These are obtained by computing the standard deviation of the estimate in question and multiplying it by an appropriate factor depending upon the probability level at which one wishes to fix the confidence range.

27. For the estimates obtained from the probit-height regression line which, as noted above, appear to be the "best" estimates obtainable from the data, the respective 99% confidence intervals are:

1% Height: 1.32 - 1.62
50% Height: 2.49 - 2.59
99% Height: 3.46 - 3.74

28. The data obtained in the Bruceton tests were analyzed in the usual manner as described in reference 1. Estimates of $\bar{X}$ (50% PT.) and σ (std. dev.) were first obtained from each of the four Brucetons consisting of 200 trials each. These tests were then broken down into groups of 100 and finally into groups of 50 and estimates of $\bar{X}$ and σ again computed, so that 28 sets of estimates were obtained in all. These are tabulated below together with corresponding estimates of the 1% and 99% points. Since they are relatively familiar, details of the calculations are not included in this report.

1/4 Inch Intervals

No. Trials	$\bar{X}$	σ	1% Pt.	99% Pt.
200	2.55	.32	1.81	3.29
100	2.54	.33	1.77	3.31
50	2.54	.33	1.77	3.31
50	2.54	.33	1.77	3.31
100	2.56	.32	1.82	3.30
50	2.58	.24	2.02	3.14
50	2.53	.39	1.62	3.44

1/2 Inch Intervals

No. Trials	$\bar{X}$	σ	1% Pt.	99% Pt.
200	2.60	.42	1.62	3.58
100	2.64	.48	1.52	3.76
50	2.71	.66	1.17	4.25
50	2.57	.27	1.94	3.20
100	2.57	.34	1.78	3.36
50	2.63	.30	1.93	3.33
50	2.51	.35	1.70	3.32

In the tabulation of the results obtained using logarithmically spaced intervals, σ will be given in logarithm units since it has significance only when thus expressed.

.05 Logarithm Intervals

No. Trials	$\bar{X}$	σ (log.)	1% Pt.	99% Pt.
200	2.50	.087	1.57	3.97
100	2.50	.078	1.65	3.79
50	2.53	.090	1.56	4.09
50	2.47	.068	1.72	3.56
100	2.47	.096	1.48	4.12
50	2.43	.127	1.23	4.80
50	2.48	.061	1.79	3.44

.10 Logarithm Intervals

No. Trials	$\bar{X}$	σ (log.)	1% Pt.	99% Pt.
200	2.45	.084	1.57	3.85
100	2.42	.104	1.39	4.23
50	2.39	.099	1.41	4.06
50	2.45	.107	1.38	4.36
100	2.49	.063	1.77	3.49
50	2.43	.066	1.71	3.47
50	2.55	.057	1.88	3.46

29. A graphical picture of the distribution of these estimates and their degree of conformity with the corresponding estimates obtained by the probit analysis of the entire data on the arithmetic scale is presented in Figures 3, 4 and 5. On these graphs drop-heights are measured along the vertical axis and estimates obtained from the various tests are represented by horizontal lines drawn at the appropriate height. The solid line drawn across the entire length of the graph represents the probit estimate of the percentage point in question and the two broken lines which bracket it indicate its confidence interval. The

shorter horizontal lines represent Bruceton estimates involving various numbers of trials as indicated by the legend at the bottom of the graph. Each graph has been divided into four sections by vertical lines, each section containing the estimates obtained from tests conducted with a given interval size as specified on the 200-trial line in each case. The shorter lines in each section represent estimates obtained through the subtests into which the 200-trial tests were broken as described previously.

30. A glance at the graphs shows that the Bruceton estimates are subject to a considerable dispersion and that many of them fall outside the 99% confidence ranges for the probit estimates. As would be expected, this variation is much less marked in the case of the 50% point than for the extremes. Focusing attention on Figure 3, there seems to be a definite tendency for estimates of the 50% point obtained using arithmetic intervals to be above the "true" value and for those obtained using logarithmic intervals to be below it. It may also be noted that on the basis of the number of estimates of the 50% point falling within the probit range, the 1/4-inch interval was considerably superior to any of the others. It should perhaps be noted that had a similar comparison been made using the logarithmic probit confidence range, the .05-log interval would have placed the most estimates within that range, with the 1/4-inch interval close behind.

31. The results of the above comparison seem to be in accord with theoretical considerations which indicate that small interval sizes (about .5) are best for estimating the 50% point.

32. Perhaps the most striking fact indicated by the graphs is that whereas the 1/4-inch interval made the best showing for the 50% point, placing all seven estimates within the confidence range, it fared worse than any of the other interval sizes in estimation of the 1% and 99% points, failing to place a single estimate within the confidence intervals. Since estimates of extreme percentage points are obtained by adding or subtracting certain multiples of the standard deviation, σ, to the mean or 50% point, $\bar{X}$, this can only mean that this interval size, while excellent for estimating $\bar{X}$, was very poor for estimating σ. This is again in accord with theoretical considerations, which indicate that small interval sizes are inefficient for estimating σ.

33. The practical fact which must therefore be faced in sensitivity testing is that $\bar{X}$ and σ cannot be estimated with optimum accuracy by a single test. In recognition of

this fact the Bruceton test is classified under two separate headings in reference 4. They are referred to as the Up and Down-Small Interval Size and the Up and Down-Large Interval Size. The former is recommended for estimation of only the 50% point, whereas the latter is recommended for the simultaneous estimation of more than one of the 10%, 50% and 90% points.

34. It seems fairly evident, however, that an even better procedure for the estimation of the 50% point and one or more extreme points would be to resort to two tests, one to estimate $\bar{X}$ and the other to estimate σ. This alternative is, of course, limited by the amount of experimental material one can afford to expend, but it would seem to merit serious consideration wherever the availability of expendable material permits.

35. For the data under present consideration the estimate of σ from the arithmetic probit analysis is .46 inches. Reference to the Bruceton estimates tabulated above shows that the estimates obtained in the 1/4 inch tests are uniformly too low. Unfortunately the 1/2 inch tests showed a distressing randomness in the estimation of σ but two rather good estimates were obtained, i.e., .42 for the entire 200-trial sequence and .48 for one of its 100-trial subsequence. In the case of the logarithmic Bruceton tests the best estimates of σ (as compared with the log probit estimates) were those obtained from 200 and 100-trial tests. There seem to be experimental indications that more trials are necessary for the accurate estimation of σ than for $\bar{X}$. This would mean that if, as recommended above, a prescribed number of trials are to be divided into two sets, one using small interval sizes to estimate $\bar{X}$, and the other using larger interval sizes to estimate σ, a greater proportion of the trials should be included in the latter test than in the former. The optimum ratio for this proportion would seem to merit further study.

36. Taken as a whole the picture of the Bruceton estimates is not a particularly happy one. Figures 4 and 5 show an alarming spread among the estimates of extreme percentage points. Large sample size seems to give no guarantee of success. Of the 200 trial tests only one of the four estimates fell within the probit confidence interval for the 99% point and two of the four for the 1% point. The use of many of the estimates to specify extreme percentage firing points would have proved highly misleading. It might be mentioned, however, that if the confidence intervals of the log probit analysis had been used, each of the 200-trial log Bruceton estimates would have fallen within the required limits for both the 1% and 99% points, although

most of the estimates obtained from a lesser number of trials would have fallen outside the confidence interval.

37. If one looks for a pattern in the dispersion of the Bruceton estimates it is seen from the graphs that for arithmetic intervals the estimates tend to be too low in the case of the 99% point and too high in the case of the 1% point. This state of affairs is undoubtedly due to a consistent underestimation of σ. In the case of the logarithmically spaced tests the estimates tend to be high at both extremes when compared with the arithmetic estimates. This state of affairs would rather be expected, as previously noted, and would not have prevailed if the logarithmic probit confidence interval had been employed on the graph.

38. It should, of course, be noted that confidence intervals can be computed for the Bruceton estimates as well as for the probit estimates and that, if these were included on the graphs, the confidence intervals surrounding some of the Bruceton estimates which lie outside the probit range would overlap that range.

39. This leads to the point that no estimate of any percentage point should be given without also specifying either a confidence interval or the standard deviation of the estimate from which a confidence interval can be computed. Without this additional information the person to whom the estimate is furnished has no idea of how much reliance may be placed upon it. The calculations involved in computing the confidence intervals for the Bruceton estimates are described in reference 1, pgs. 20-22, and in reference 3, pg. 97. As an illustration, the estimates with their corresponding 99% confidence intervals obtained from the 200-trial 1/2 in. Bruceton test conducted as part of the present study are as follows:

1% point	1.62 ± .37
50% point	2.60 ± .11
99% point	3.58 ± .37

Note that although the estimate of the 50% point is considerably above the probit estimate of 2.53, its confidence interval includes that value, extending from 2.49 to 2.71.

40. It has been shown theoretically that the reliability of the Bruceton method decreases rather rapidly with the distance of the point estimated from the 50% point. For example, it has been shown (reference 4, page 91) that the method is only about half as accurate for estimating the 10% point as for estimating the 25% point. In this connection it should be noted that in reference 4 the Bruceton

method is not recommended under any conditions for estimating points outside the 10%-90% range. For precise estimates of more extreme points other methods should be resorted to.

41. The question of arithmetic vs. logarithm intervals was further investigated by application of the χ^2 test to the separate 200-trial Bruceton tests as suggested in reference 1, pages 29-33. The calculations are shown on Data Sheet 4 and the results are as follows:

Interval	Chi-Square Value	Probability Level
1/4 inch	$\chi_2^2 = 1.40$	.50
1/2 inch	$\chi_1^2 = .68$	.43
.05 (log)	$\chi_3^2 = 2.86$	.42
.10 (log)	$\chi_1^2 = .95$	.34

42. The subscript on each of these χ^2's represents its number of degrees of freedom, and since this number varies the χ^2 values cannot be compared directly. For this reason the probability of obtaining a larger χ^2 value has been given in each case. Since high probability indicates good fit, the evidence again points, as in the case of the χ^2 analysis of the entire set of data, to the fact that arithmetically spaced intervals are at least as good as, if not somewhat better than, logarithmic intervals for the Bruceton test.

43. It might be well to point out that, entirely aside from theoretical considerations of normality, logarithmic intervals suffer a practical limitation which restricts their usefulness. To elucidate this point let it be recalled that in calculating the mean for a set of Bruceton data the test heights are coded to the integers 0, 1, ... etc. Thus if the testing heights are denoted by X_i, the lowest testing height by X_0, and the interval size by d, then the coded heights, h_i, are given by:

$$h_i = \frac{X_i - X_0}{d}$$

so that $h_0 = 0$, $h_1 = 1$, etc. A weighted mean, $\ddot{H}$, is then computed and $\ddot{X}$ is obtained, using the relationship

$$\ddot{X} = X_0 + d\ddot{H}$$

It is from this coding that the simplicity of the Bruceton analysis is derived and for arithmetically spaced intervals it is beyond criticism. In the case of logarithmic intervals the situation is complicated by the fact that in attempting to space the intervals equally on a logarithmic scale one is limited by the accuracy with which the drop testing device is calibrated. This means that, having chosen a certain logarithmic interval which one wishes to use in spacing the testing heights, one is forced in each case to choose the calibrated testing height which is nearest to the correct height and consequently the testing heights are not really equally spaced. The mathematical consequence of this is that the coded heights, 0, 1, 2, .., do not correspond to the actual heights and an additional error is therefore introduced into the Bruceton calculations. For example, in the 200-trial Bruceton test at logarithmic intervals of .05, the lowest actual testing height was 1.80 inches and the highest was 3.55 inches. This actual logarithm of these heights is to three decimal places, .255 and .550. Using these values we have:

$$h_6 = \frac{.550 - .255}{.05} = 5.9$$

whereas the integral value, $h_6 = 6$, was actually used in the Bruceton calculations. The error introduced in this way could have been avoided by using the actual testing heights in calculating m and σ, but in so doing one of the principal virtues of the Bruceton analysis, namely simplicity of calculation, would have been lost. It should of course be emphasized that the limitation just described is not intrinsic to the logarithmic method itself but, rather, that it arises in connection with its use with a particular drop-testing apparatus.

44. Before concluding the discussion of arithmetic vs. logarithmic drop heights one more point which has been raised in that connection will be considered. This is that in using arithmetic heights a negative drop height is occasionally estimated to correspond to a low explosion percentage.

45. It is of course evident at the outset that whenever the normal function is used to predict the height corresponding to a given explosion percentage this height will be negative if the given percentage is small enough. This is a consequence of the fact that the cumulative normal curve or ogive is, at its lower tail, asymptotic to the horizontal axis, lying entirely above it, and therefore the extreme portions of this section of the curve would predict positive explosion percentages for zero height or for any negative height. As a matter of fact the same difficulty is met at the other end of the distribution where the curve is asymptotic to a line parallel to the horizontal axis and one unit above it. Here a positive percentage of misfires will be predicted for any drop height however large. All of this simply means that

whereas the normal curve may be an excellent approximation to the true explosion curve over the central portion of its range, corresponding roughly to $m \pm 3$, it cannot be used outside that range and this will cause no difficulty in most practical situations.

46. If one uses a logarithmic scale the difficulties encountered at the lower end of the distribution seem to be avoided since a height predicted from the fitted normal curve will always be expressed in logarithmic units and hence the corresponding arithmetic height must always be positive. The difficulties at the other end of the range remain.

47. The device of using the logarithmic scale simply to avoid the occasional estimation of a negative drop height seems, however, to be an artificial mode of escape and its choice on this ground alone seems hardly justified. The main consideration would seem to be the choice of a function, whether height, log height, or possibly some other one, which best approximates the normal distribution. Having chosen the function which best meets this criterion, an occasional encounter with a negative height should be looked upon simply as a consequence of the use of the normal distribution and should cause no undue alarm.

48. It might be noted at this point that since the logarithmic scale has been by no means proved not to be the true distribution, its use in certain situations for estimating percentage firing points at the upper extreme merits consideration in view of the previously noted tendency for logarithmic estimates to be higher than the arithmetic ones. For this reason a logarithmic estimate of a high percentage point might be said to be somewhat safer than an arithmetic one since the probability of underestimation is smaller. For the same reason an arithmetic estimate might be regarded as safer at the lower extreme since at that end of the range overestimation of a percentage firing point is to be avoided for reasons of safety.

The Mk 120 Type Primer and the Primer Mk 102
(Stab Initiated Primers)

Test Procedure

49. The statistical procedure used in the study of sensitivity tests of the Mk 120 type primer was identical with that used in the case of the Mk 101. The results obtained from these tests will be detailed and interpreted below.

Results

50. Since only 800 primers were available for testing purposes, the scope of the testing program was necessarily reduced as compared with the Mk 101 primer, but the same scheme was followed. In this case only two Bruceton tests were conducted, each consisting of 200 trials and at intervals of 1/2 inch and .04 (log) respectively. After the completion of these two Bruceton tests the remainder of the primers was expended at various levels in order to obtain as complete a rundown as possible. As will be seen from column 4 of Data Sheet 5, a 100% experimental point was reached with 40 trials at 8.50 inches, but the lowest empirical percentage obtained was 2% in 50 trials at 1.50 inches.

51. The probit analysis of the entire set of data did not produce results which were nearly as reassuring as far as the criterion of normality was concerned, as was the case with the Mk 101 primer. The plotted probits are shown for height on Figure 6 and for log height on Figure 7. In both cases, although a general linear trend is evidenced, there is also considerable scatter.

52. The probit analysis was carried through two cycles in both cases. The graphs show the original line fitted by observation and the two adjusted lines. The line produced by the second cycle in the case of log height was practically indistinguishable from the previous line and hence has not been shown on the graph.

53. The calculations involved in the probit analysis are shown on Data Sheets 5 and 6. The χ^2 values for both stages of the adjusted line are shown below:

	Arithmetic	Logarithmic
1st cycle	$\chi^2_{14} = 32.163$	$\chi^2_{14} = 34.290$
2nd cycle	$\chi^2_{14} = 26.747$	$\chi^2_{14} = 31.209$

54. These values are well above the expected chance levels and hence the hypothesis of normality is definitely under suspicion in both cases. As in the case of the Mk 101 type primer, some superiority for the arithmetic scale was indicated.

55. In some cases a large χ^2 value may be misleading in view of the fact that it contains unusually large contributions from the ends of the range where the probabilities are very high or very low, and in such a case the

fit may be actually better than the χ^2 value would indicate. In these cases a more accurate picture may sometimes be obtained by grouping several levels together at each end of the range. This was carried out for the arithmetic scale, the calculations being shown on Data Sheet 7, and a slight improvement was shown. However, the value $\chi^2_9 = 15.21$ is still a rather improbable one under the hypothesis of normality and this hypothesis, while not to be rejected, is certainly not strongly supported.

56. In the probit analysis, departures from normality as indicated by a large χ^2 value are allowed for by applying a heterogeneity factor to the variances of the estimates. This factor has the effect of increasing the size of the confidence interval for each estimate. In other words, the net effect of considerable departures from normality is to decrease the precision with which estimates of the various percentage firing points can be specified. The use of the heterogeneity factor will be illustrated presently.

57. The equation of the fitted probit regression line as obtained from the second cycle of calculations on the arithmetic scale is

$$Y = 2.34 + .59X$$

where, as before, X represents the drop height and Y the corresponding probit. Using this equation the following heights were estimated:

1% Height : .56 inches
50% Height : 4.51 inches
99% Height : 8.46 inches

The variances of the three estimates listed above are .0838, .0084, and .0765, respectively. However, because of poor fit these quantities must be corrected by the heterogeneity factor as stated above. This factor is obtained by dividing the χ^2 value obtained by its number of degrees of freedom. Using the χ^2 value which was obtained by grouping several levels together we have:

$$\frac{\chi^2_9}{9} = \frac{15.21}{9} = 1.69$$

If the variances listed above are multiplied by the factor they become .1416, .0142, and .1293, respectively. The respective standard errors are obtained by extracting the square roots of these numbers, and these standard errors are then multiplied by a factor obtained from a table of

Student's "t" to obtain the following confidence intervals:

1% Height : 0 - 1.80
50% Height : 4.12 - 4.90
99% Height : 7.29 - 9.63

58. The confidence interval for the 1% height actually includes negative heights but only the positive portion of the range has been tabulated.

59. The first and most obvious remark to be made about these confidence intervals is that they are very wide. This becomes even more evident if they are compared with the corresponding intervals obtained in the tests of the Mk 101 type primer and listed on page 6 of this report. The upshot of this is that the experimental material used in the study of the Mk 120 type primer behaved in such a way that it is impossible to confine estimates of its percentage firing points to reasonably small confidence intervals even through the use of the relatively elaborate probit analysis.

60. The results of the various Bruceton tests and subtests are tabulated below:

1/2 inch intervals

No. trials	$\bar{\bar{X}}$	σ	1% Pt.	99% Pt.
200	4.86	1.51	1.35	8.37
100	4.60	1.47	1.18	8.02
50	4.42	.95	2.21	6.63
50	4.79	1.76	.70	8.88
100	5.12	1.16	2.42	7.82
50	5.10	.73	3.40	6.80
50	5.12	1.59	1.42	8.82

.04 (log) intervals

No. trials	$\bar{\bar{X}}$	σ(log)	1% Pt.	99% Pt.
200	4.48	.220	1.38	14.55
100	4.81	.189	1.75	13.24
50	5.18	.098	3.06	8.75
50	4.52	.237	1.27	16.07
100	4.16	.165	1.72	10.07
50	4.06	.113	2.22	7.45
50	4.30	.205	1.43	12.88

61. As would be expected from experimental material of this kind, an extremely wide range of variation was exhibited. For instance, it will be noted that estimates of the 1% point as obtained from the 1/2 in. Bruceton tests vary from .70 inches to 3.40 inches, whereas estimates of the 99% pt. as obtained from the .04 log test vary from 7.45 to 14.55 inches. Needless to say, little reliance could be placed on any single Bruceton estimate obtained for primers of this sort.

62. The confidence ranges of the probit estimates were so wide and the variation of the Bruceton estimates so extreme that there seemed little point in illustrating that variation graphically as was done with the test results of the 101 primer. If the same scale was used as in the previous graphs even the confidence intervals of 1% and 99% probit estimates could not be contained on a single sheet and to compress the scale would only be misleading.

63. The χ^2 tests for the two individual 200-trial Brucetons were run as was done in the case of the Mk 101. The calculations are shown on Data Sheet 8 and the results are shown below:

Interval	Chi-square value	Probability Level
1/2 inch	$\chi^2_5 = 4.96$	.43
.04 (log)	$\chi^2_8 = 13.50$	.10

64. The value obtained for the arithmetic intervals seems to contradict the poor fit indicated by the probit analysis of the entire rundown. However, since this value is derived from one-fourth of the total number of trials and concentrated in the neighborhood of the 50% point, it is not adjudged to be as reliable an indicator of the true situation as the probit χ^2 value. Comparison of the two χ^2 values listed above again indicates superiority for arithmetically spaced intervals for the Bruceton test.

65. Before concluding the remarks on the Mk 120 type primer the results of a set of calculations carried out prior to the initiation of the present study should perhaps be mentioned. There was available for analysis a set of data obtained from a rundown test of Mk 102 primers which had been conducted in September 1944. The priming mixture contained in the primers was mercury fulminate Pom Pom No. 74. The rundown was complete, consisting of 100 trials at each of 16 levels spaced 1/4 inch apart.

66. The data were analyzed by the probit method. The probit curves are shown on Figures 8 and 9 and the calculations on Data Sheets 9, 10 and 11. A glance at the explosion percentages shown in Column 3 of Data Sheet 9 shows a curious slowness of tailing off at the upper end of the range. Nearly half of the total range of test heights is included in the 90%-100% explosion percentage interval. This factor undoubtedly affected the statistical results, as will be seen.

67. The χ^2 values obtained after a single cycle of calculations were as follows:

Height	Log Height
$\chi^2_{13} = 35.347$	$\chi^2_{13} = 14.951$

68. The calculations for height were carried through a second cycle but the fit was even poorer, a possibility previously alluded to. A third χ^2 value for height was calculated by grouping extreme intervals but no improvement in fit was noted.

69. Thus the two χ^2 values shown above seem to contradict the evidence obtained in the main body of this study in that the logarithmic scale seems to be definitely superior to the arithmetic as far as goodness-of-fit to the normal distribution is concerned. However, a study of the graphs strongly indicates that the poor fit indicated for height is in the main caused by the last few points at the top of the probit graph; that is, the points corresponding to the probits determined by the unusually long tail of the distribution. Use of the logarithmic scale brings about a contraction of the horizontal height scale at its upper end, thus bringing the points plotted at the tail closer to the line determined by the remaining points.

70. The result obtained in this case is therefore not thought to contradict the conclusions previously stated concerning the relative value of arithmetic vs. logarithmic intervals for the testing of primers in general. In this case the unusually long tail at the upper end of the distribution was thought to reflect inferior quality in the primers under test. Logarithmic tests would therefore seem to merit serious consideration whenever material of this kind is under test.

Conclusions

71. The following general conclusions appear applicable

to the tests which have been described and discussed in this report:

a. The probit analysis should replace the Bruceton whenever the exact specification of extreme percentage points is required. Attention is again invited to references 3 and 6 in this connection.

b. Whenever possible, when using the Bruceton method, separate tests using different interval sizes should be used to estimate and

c. In any case, Bruceton tests should not be used to estimate points outside the 10%-90% range.

d. No marked superiority for either arithmetic or logarithmic intervals has been disclosed. Either may be used as seems appropriate.

e. Appropriate confidence intervals should be computed and furnished with all estimates.

72. Some qualification should be applied to the above conclusions, however, in those cases where primers exhibit the erratic performance noted for the Mk 102 and Mk 120 type described above. The use of the Bruceton test in evaluating primers which show erratic behavior would seem to be open to serious question. The probit analysis with appropriately stated confidence intervals would seem to be a reasonable alternative.

Suggestions for Further Study

73. In view of the seeming unreliability of the Mk 120 and Mk 102 types of primers tested in the course of this study, further tests along the lines suggested in this report using a large group of Mk 102 and Mk 120 types of primers manufactured under carefully supervised conditions to insure uniformity would be of value.

74. Experimental testing of other "staircase" methods discussed in reference 4 would throw further light on tests for small samples that might be used to supplant the Bruceton. In most cases the alternative methods are considerably more complex in both testing procedure and subsequent analysis than is the case with the Bruceton and they are less flexible, allowing the estimation of only one percentage point for each test. Whether these disadvantages are offset by greater accuracy would be worth investigation.

75. Finally, in an attempt to diagnose the cause of apparent inconsistencies of primer firing under drop-test conditions, a critical review of the entire drop-test procedure would seem to be in order. A step in that direction has been made in reference 7, in which the problems connected with the transfer of energy from ball to primer through the medium of the firing pin are discussed from a theoretical point of view. It is pointed out in this connection that the maintenance of stable conditions with regard to this transfer of energy in the course of the test would require that the trials during a drop test be conducted from a constant height while the weights of both the ball and the firing pin were varied from trial to trial. Whether an even greater improvement could be effected by abandonment of drop-testing methods entirely in favor of testing devices of the pendulum type or timing devices in which the time interval between impulse and explosion is measured would seem to be a subject for fruitful research in the future.

Appendix A

Note on Computation of Confidence Intervals

In computing the confidence interval for a given estimate the standard deviation of the estimate is multiplied by a factor which depends upon the desired probability level. In this connection it is to be noted that these factors are not the same as those which are used in obtaining the estimates themselves, which are listed in reference 1, page 19. For example, the factor used to determine the estimates of the 99% firing point is 2.326, whereas the factor to be used in determining the 99% confidence interval of that estimate is 2.58. The correct factors to be used in computing confidence intervals at any probability level can be read from any table of normal areas and ordinates and is found in the column headed "t" opposite the entry in the column headed "   ", which is half of the desired probability. For example, to determine the correct factor for a 95% confidence interval, the value 1.96 is located in the column headed "t" opposite the value .47500 (half of .98) in the column headed "   ".*

The values used for estimating any percentage firing point may be obtained from the same table, though the values listed in reference 1 are adequate for most purposes. To obtain these values from a table of normal areas, .50 is subtracted from the required percentage, expressed as decimal, the result is located in the "   " column and the correct factor appears opposite it in the "t" column. Thus the value 2.326 used to estimate the 99% point is found by interpolating between the values 2.32 and 2.33 which appear in the "t" column opposite the values .48983 and .49010 respectively in the "   " column.

* In the probit analysis where a heterogeneity factor is applicable a table of "student's t" must be used instead of the table of normal areas and ordinates. See reference 2, page 60.

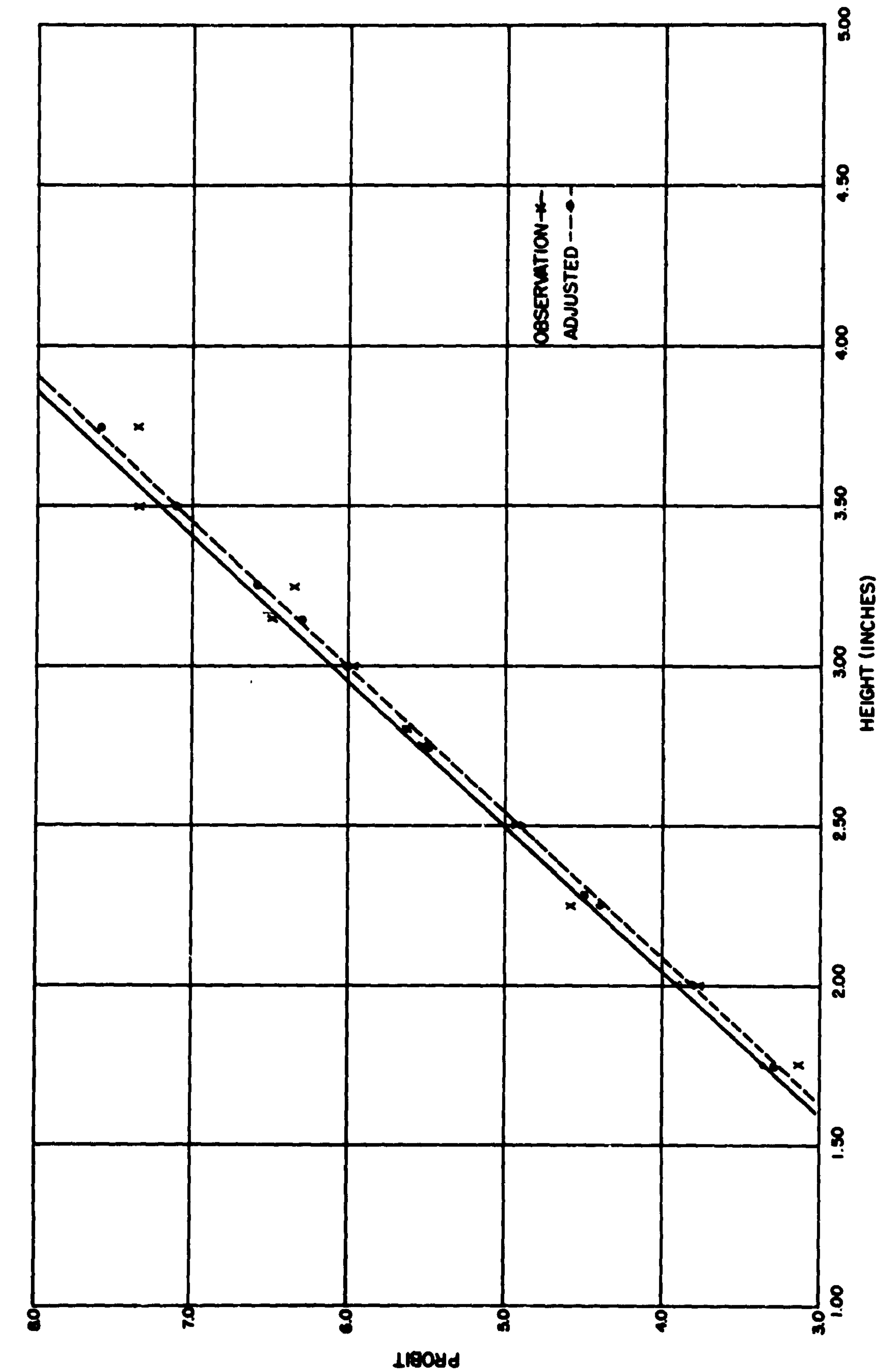

FIG. I PROBIT VS HEIGHT - MK 101 TYPE PRIMER

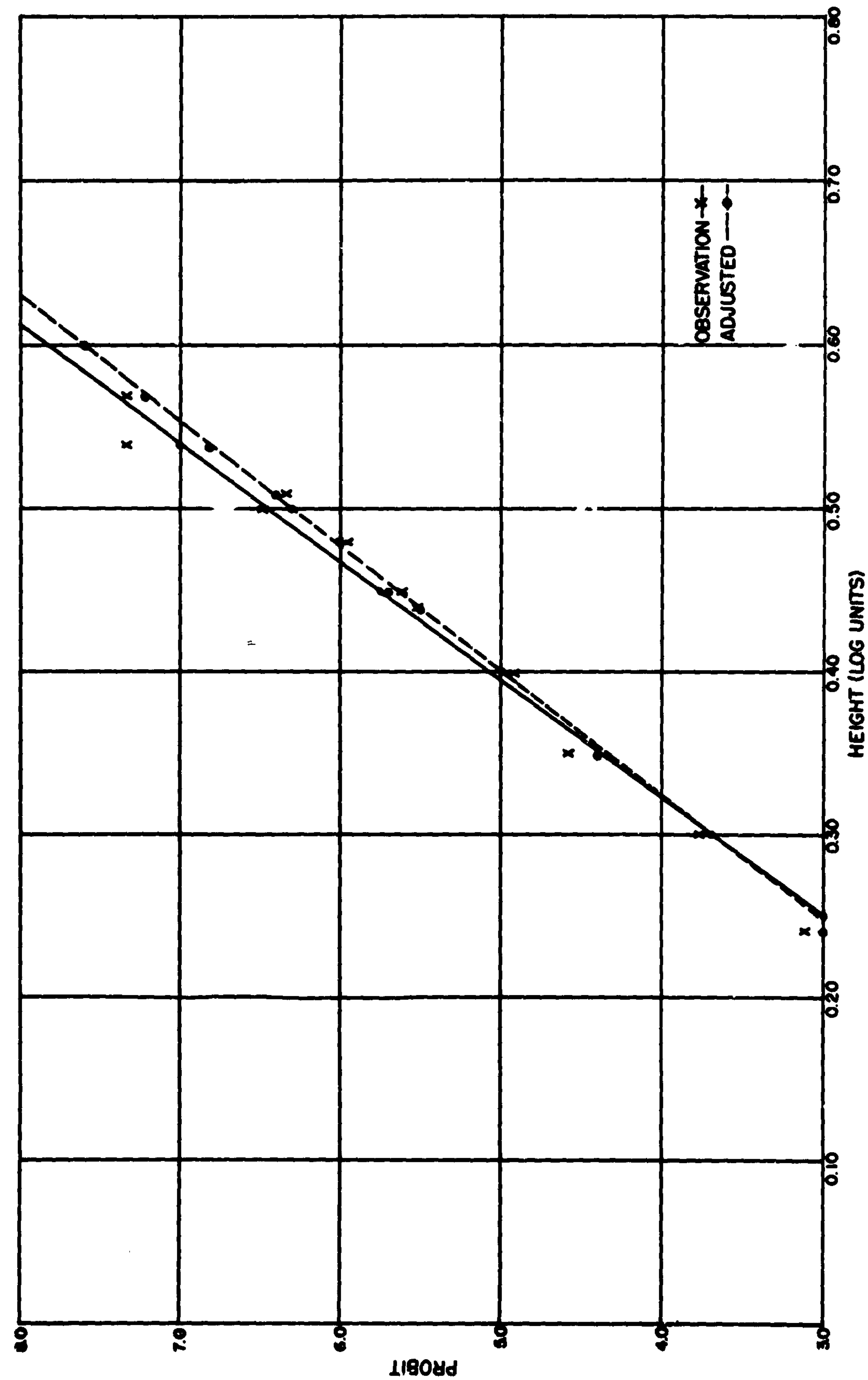

FIG. 2 PROBIT VS LOG HEIGHT - MK 101 TYPE PRIMER

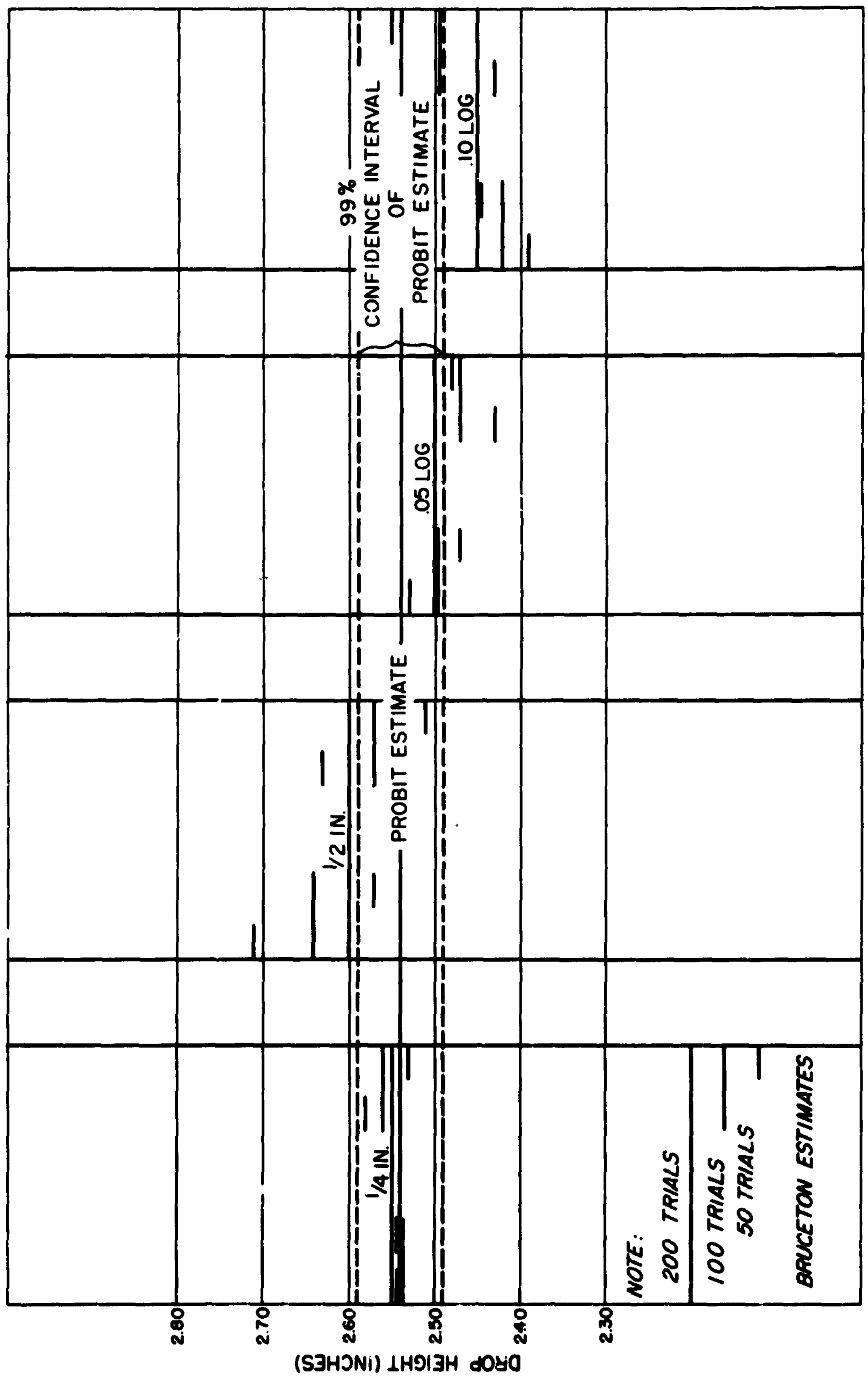

FIG. 3
PROBIT AND BRUCETON ESTIMATES OF 50% POINT
MK 101 TYPE PRIMER

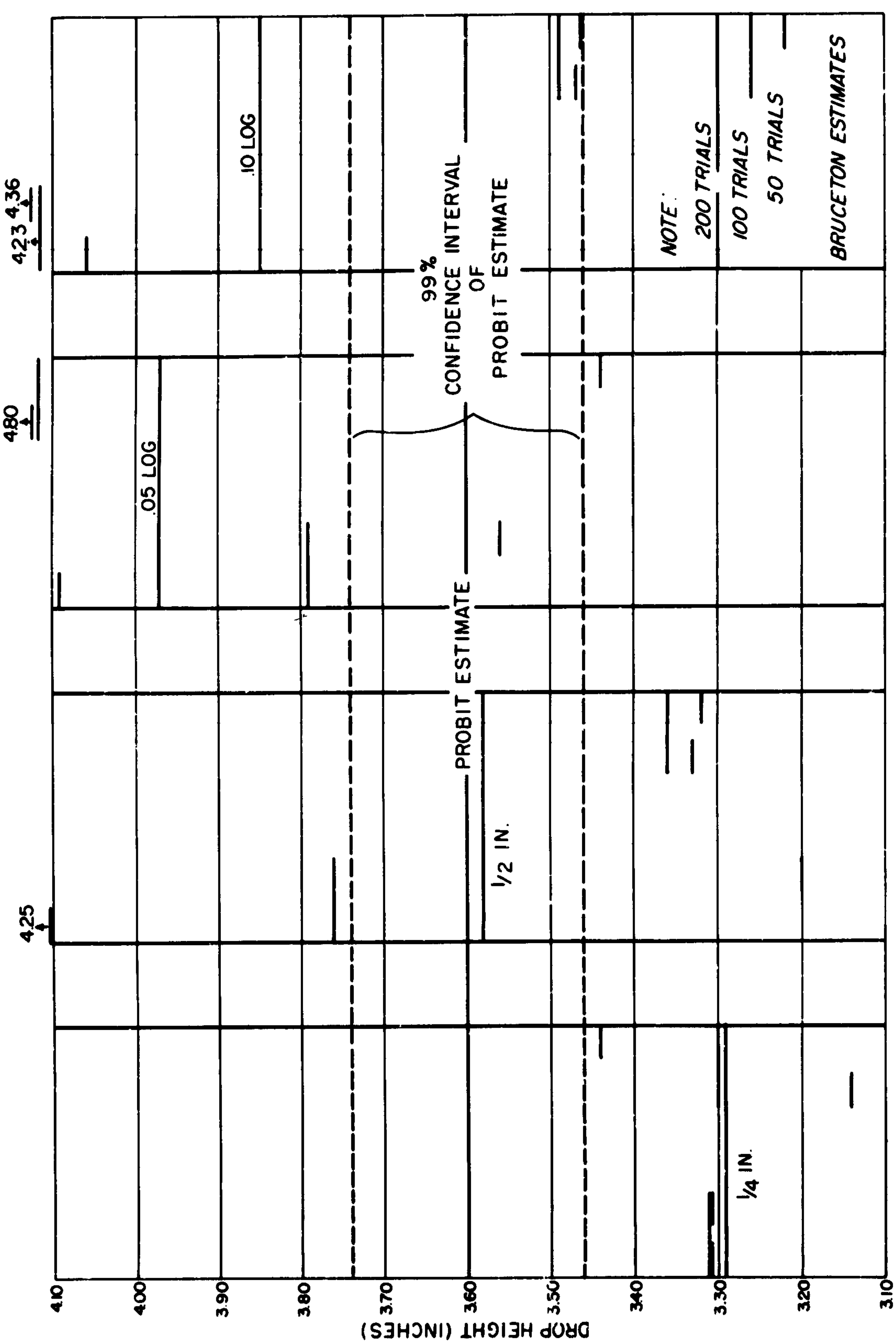

FIG. 4
PROBIT AND BRUCETON ESTIMATES OF 99% POINT
MK 101 TYPE PRIMER

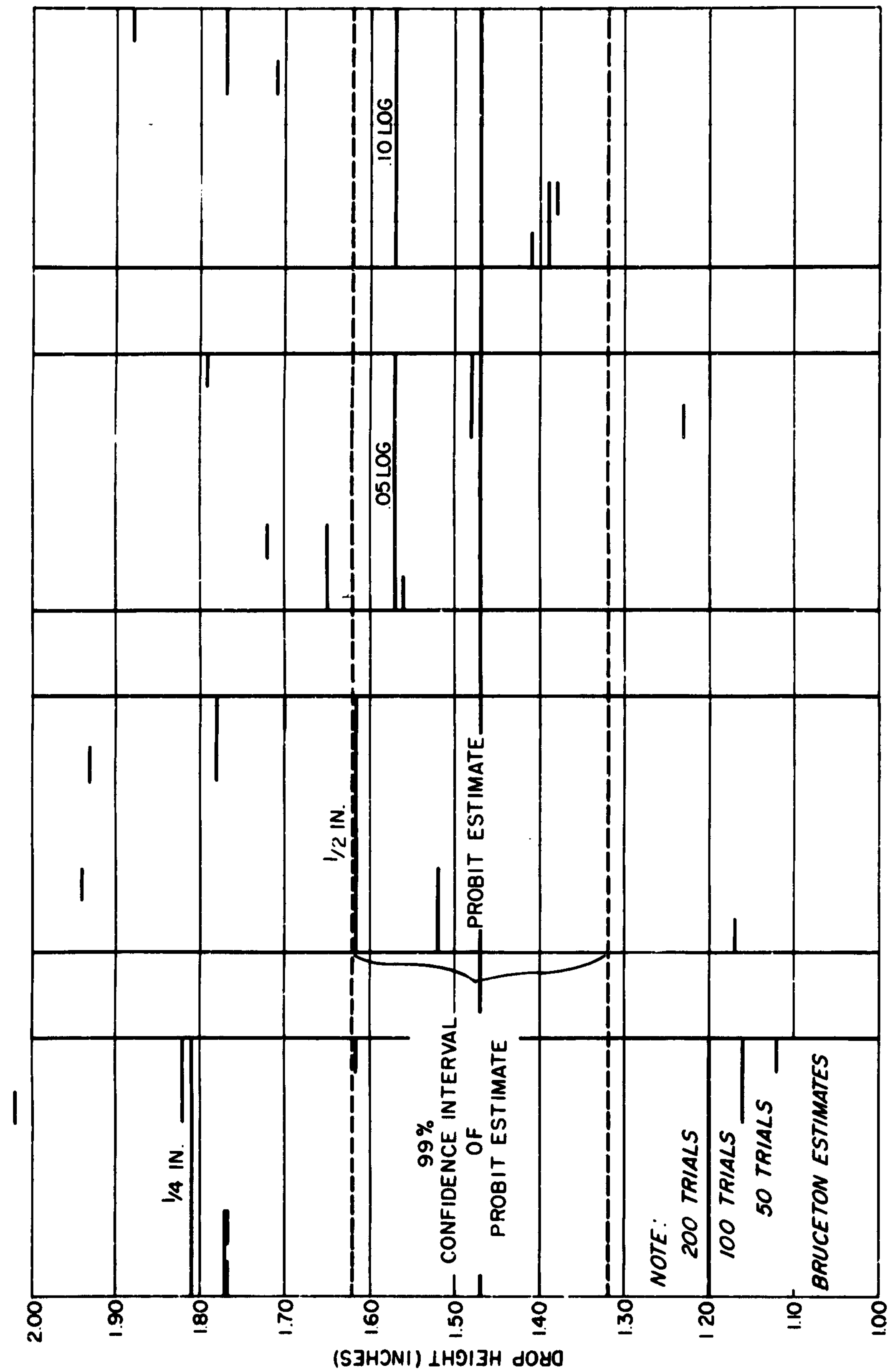

FIG. 5
PROBIT AND BRUCETON ESTIMATES OF 1% POINT
MK 101 TYPE PRIMER

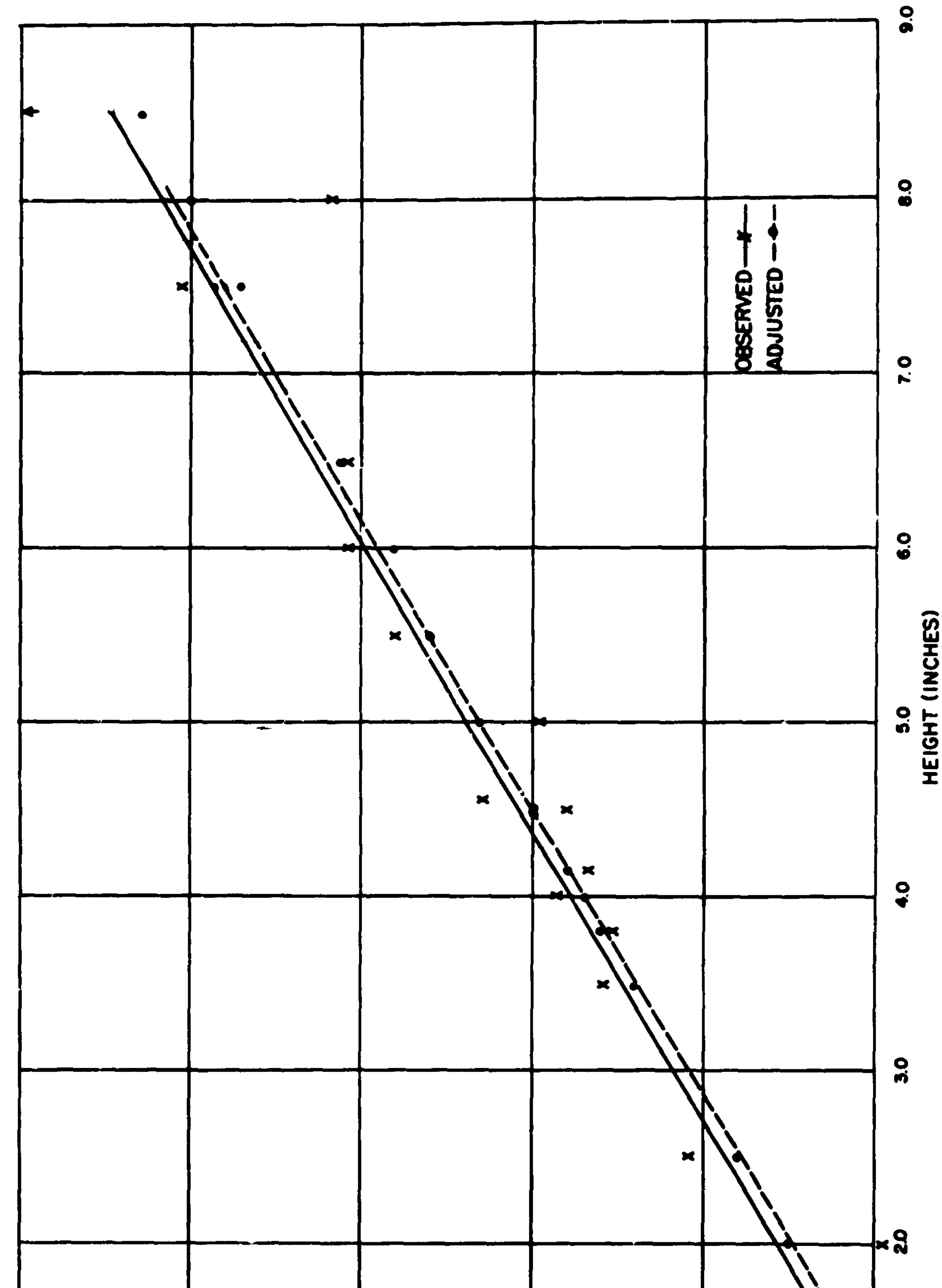

FIG. 6 PROBIT VS HEIGHT–MK 120 TYPE PRIMER

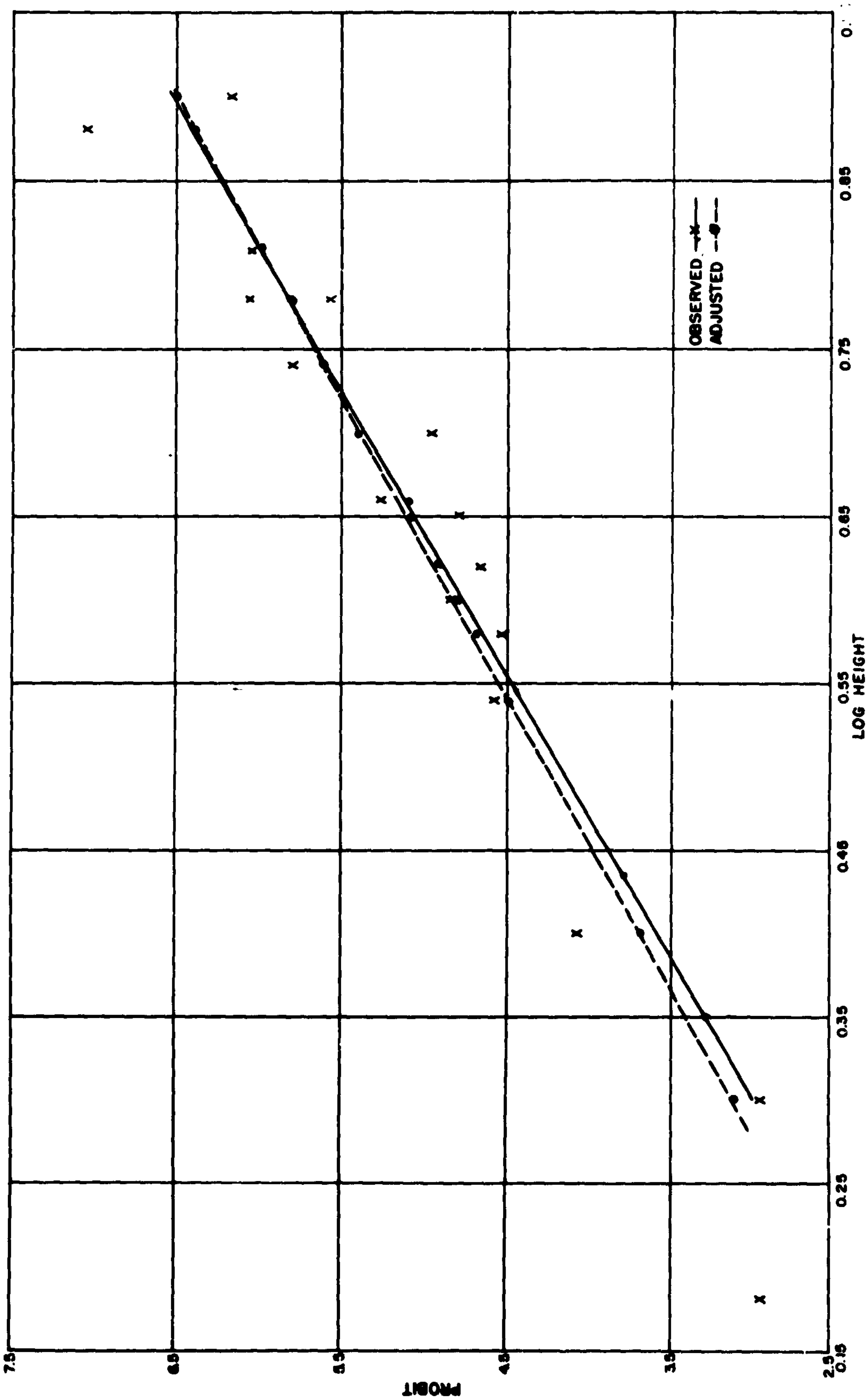

FIG. 7 PROBIT VS LOG HEIGHT — MK 120 TYPE PRIMER

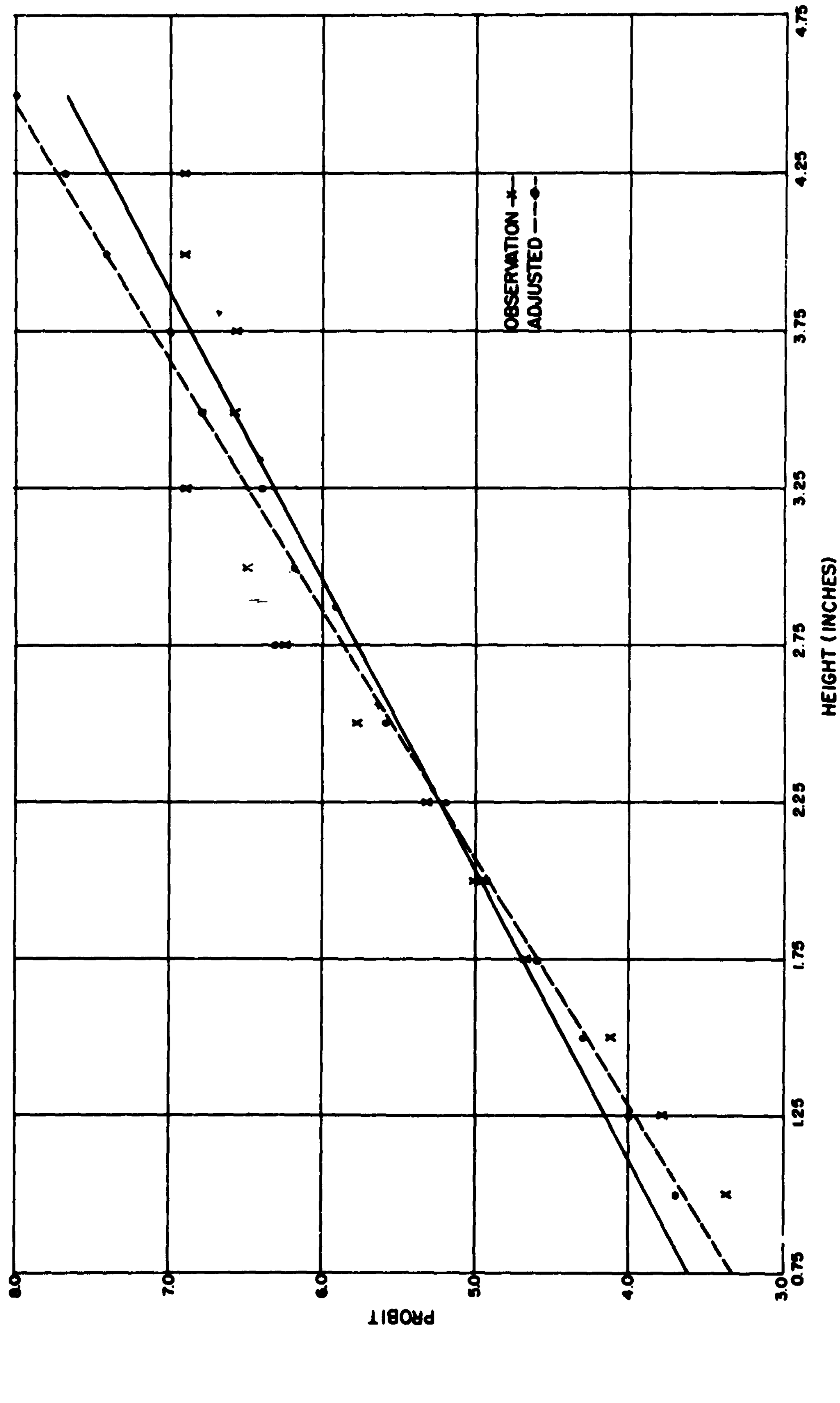

FIG. 8 PROBIT VS HEIGHT — PRIMER MK 102

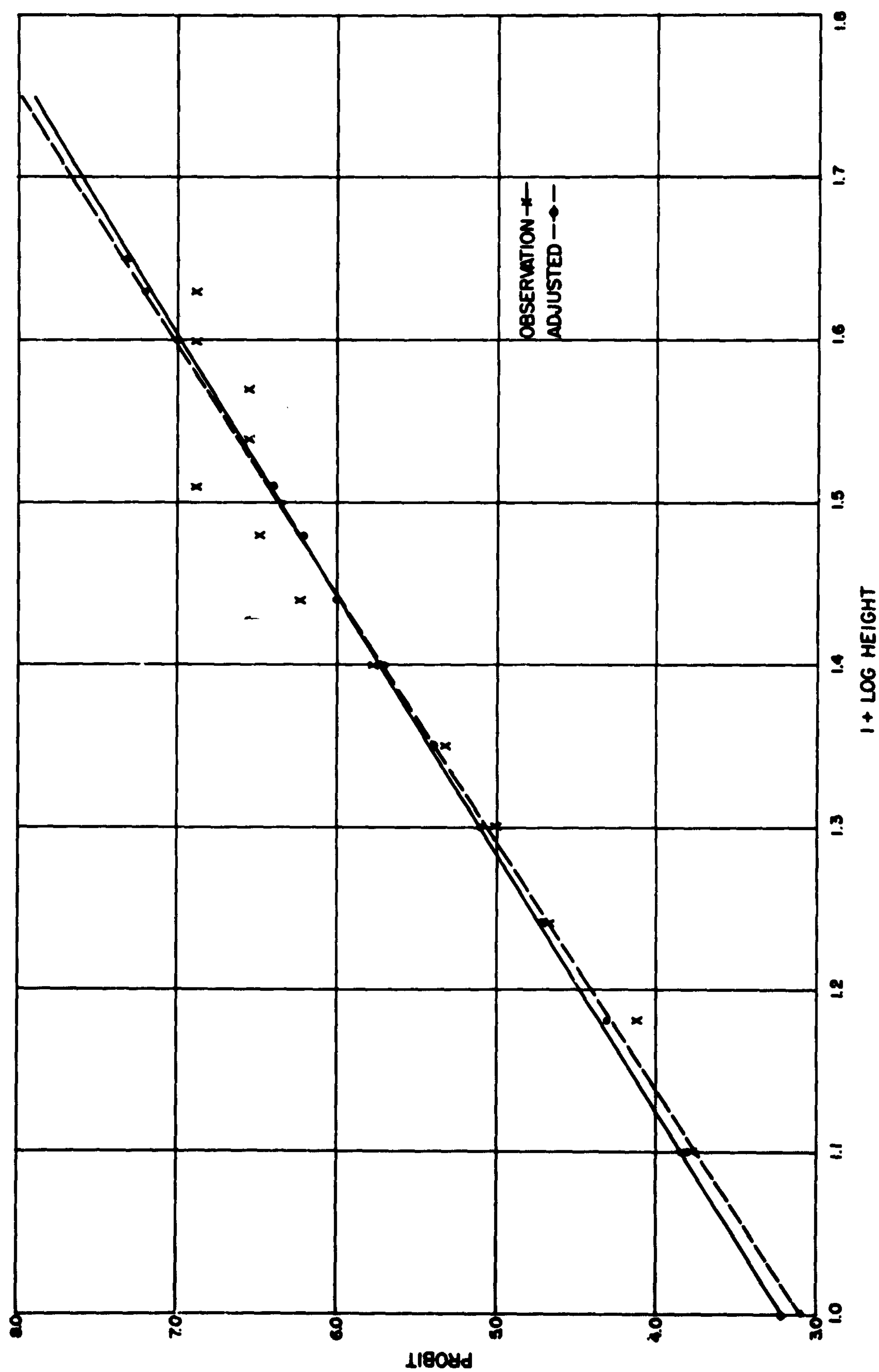

FIG. 9 PROBIT VS LOG HEIGHT PRIMER – MK 102

Probit analysis of Mk 101
1st Cycle

1 X Height	2 n No. of Trials	3 r No. of Explosions	4 p % Explosions	5 Empirical Probit	6 Y Expected Probit	7 nw Weighting Factor	8 y Working Probit	9 nwX
4.00	100	100	100		8.3	0.6	8.58	2.400
3.75	100	99	99	7.33	7.7	3.1	7.97	11.625
3.50	100	99	99	7.33	7.2	9.2	7.31	32.[illegible]
3.25	100	91	91	6.34	[illegible]	23.0	6.23	[illegible]
3.15	56	52	93	6.48	6.4	36.9	6.47	53.235
3.00	100	83	83	5.95	6.1	40.5	5.94	121.500
2.80	44	32	73	5.51	5.6	24.6	5.61	68.800
2.75	100	70	70	5.52	5.5	53.1	5.52	157.775
2.50	312	148	47	4.92	5.0	198.7	4.92	496.750
2.25	100	34	34	4.59	4.4	55.8	4.60	125.550
2.00	127	14	11	3.77	3.9	51.4	3.78	102.600
1.75	100	3	3	3.12	3.4	23.8	3.18	41.650
1.50	100	0	0	—	2.8	9.2	2.41	13.800
						515.7		1307.515

$\bar{X} = 2.5354$ $\bar{y} = 5.0007$

ΣnwX^2
3405.26675
3315.09690
90.16985

$\Sigma nwXy$
6734.2650
6538.4472
195.8178

χ^2

$b = 2.1717$ $\sigma = .46$
$Y = 5.0007 + 2.1717\,(X - 2.5354)$
$Y = -.505 + 2.172X$
$X = \dfrac{Y + .505}{2.172}$

50% pt : $X = 2.53$	$\sigma_m = .02$	99% C.I.	(2.46 – 2.58)
99% pt : $X = 3.61$	$\sigma_{99} = .06$		(3.46 – 3.76)
1% pt : $X = 1.46$	$\sigma_1 = .06$		(1.31 – 1.61)

[illegible] Analysis of [illegible] 101 Data

1st Cycle

Data Sheet 1

	8	9	10	11	12	13	14	15	16
[illegible]	y Working Probit	nwX	nwy	X Log Weight	Y	nw	y	nwX'	nwy
	8.58	2.400	5.348	.60	7.8	2.5	8.12	1.500	20.300
	7.[illegible]	11.625	21.917	.57	7.4	6.2	7.32	3.534	45.384
	7.21	32.[illegible]	67.252	.54	7.0	13.1	7.24	7.074	94.844
	6.23	[illegible]	[illegible]	.51	6.6	23.8	6.23	12.138	149.464
	6.47	53.235	109.343	.50	6.4	16.9	6.47	8.450	109.343
	5.94	121.300	240.570	.45	6.2	37.0	5.92	17.760	219.040
	5.61	[illegible]	133.026	.45	5.7	23.4	5.61	10.530	131.274
	5.52	159.775	320.712	.44	5.6	55.8	5.52	24.552	308.016
	4.92	496.750	977.604	.40	5.0	198.7	4.92	79.480	977.604
	4.60	125.550	256.680	.35	4.4	55.8	4.60	19.530	256.680
	3.78	102.600	194.232	.30	3.7	42.7	3.78	12.810	161.406
	3.18	[illegible]	75.684	.24	2.9	11.0	3.18	2.640	34.980
	2.41	13.800	22.272	.18	2.0	1.5	1.70	.270	2.550
		1307.515	2578.[illegible]			488.4		200.268	2510.885

354 $\bar{y}$ = 5.[illegible]

$\bar{X}'$ = .4100 $\bar{y}$ = 5.1410

$Snwxy$
6734.2650
6538.4472
195.8178

$Snwy^2$
13329.421
12895.910
433.[illegible]
[illegible]
[illegible]

χ^2_{11}

$SnwX'^2$
84.36360
82.11972
2.[illegible]

$SnwX'y$
1058.0579
1029.5862
28.4717

$Snwy^2$
13279.439
12908.566
370.873
360.463
= 10.410

χ^2_{11}

b = 12.6604
Y = 5.1410 + 12.6604(X−.4100)
[illegible]
[illegible]

σm = .009
$\sigma 99$ = .010
$\sigma 1$ = .011

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Probit Analysis of Mk 101
1st Cycle

1 X Height	2 n No. of Trials	3 r No. of Explosions	4 p % Explosions	5 Empirical Probit	6 Y Expected Probit	7 nw Weighting Factor	8 y Working Probit	9 nwX
4.00	100	100	100		8.3	0.6	8.58	2.400
3.75	100	99	99	7.33	7.7	3.1	7.07	11.625
3.50	100	99	99	7.33	7.2	9.2	7.31	32.200
3.25	100	91	91	6.34	6.6	23.8	6.28	77.350
3.15	56	52	93	6.48	6.4	16.9	6.47	53.235
3.00	100	83	83	5.95	6.1	40.5	5.94	121.500
2.80	44	32	73	5.61	5.6	24.6	5.61	68.880
2.75	100	70	70	5.52	5.5	58.1	5.52	159.775
2.50	312	148	47	4.92	5.0	198.7	4.92	496.750
2.25	100	34	34	4.59	4.4	55.8	4.60	125.550
2.00	127	14	11	3.77	3.9	51.4	3.78	102.800
1.75	100	3	3	3.12	3.4	23.8	3.18	41.650
1.50	100	0	0	—	2.8	9.2	2.41	13.800
						515.7		1307.515

$\bar{X} = 2.5354$ $\bar{y} = 5.0007$

$SnwX^2$	SnwXy	
3405.26675	6734.2650	1
3315.09690	6538.4472	1
90.16985	195.8178	

χ^2_{11}

$b = 2.1717 \quad \sigma = .46$

$Y = 5.0007 + 2.1717\,(X - 2.5354)$

$Y = -.505 + 2.172X$

$X = \dfrac{Y + .505}{2.172}$

50% pt : $X = 2.53$	$\sigma m = .02$	99% C.I.	(2.48 – 2.58
99% pt : $X = 3.61$	$\sigma 99 = .06$		(3.46 – 3.76
1% pt : $X = 1.46$	$\sigma 1 = .06$		(1.31 – 1.61

Probit Analysis of Mk 101 Data

1st Cycle

Data Sheet 1

7	8	9	10	11	12	13	14	15	16
nw ghting ctor	y Working Probit	nwX	nwy	X' Log Height	Y	nw	y	nwX'	nwy
.6	8.58	2.400	5.148	.60	7.8	2.5	8.12	1.500	20.300
.1	7.07	11.625	21.917	.57	7.4	6.2	7.32	3.534	45.384
.2	7.31	32.200	67.252	.54	7.0	13.1	7.24	7.074	94.844
.8	6.28	77.350	149.464	.51	6.6	23.8	6.28	12.138	149.464
.9	6.47	53.235	109.343	.50	6.4	16.9	6.47	8.450	109.343
.5	5.94	121.500	240.570	.48	6.2	37.0	5.92	17.760	219.040
.6	5.61	68.880	138.006	.45	5.7	23.4	5.61	10.530	131.274
.1	5.52	159.775	320.712	.44	5.6	55.3	5.52	24.552	308.016
.7	4.92	496.750	977.604	.40	5.0	198.7	4.92	79.480	977.604
.8	4.60	125.550	256.680	.35	4.4	55.8	4.60	19.530	256.680
.4	3.78	102.800	194.292	.30	3.7	42.7	3.78	12.810	161.406
.8	3.18	41.650	75.684	.24	2.9	11.0	3.18	2.640	34.980
.2	2.41	13.800	22.172	.18	2.0	1.5	1.70	.270	2.550
.7		1307.515	2578.844			488.4		200.268	2510.885

.5354 $\bar{y} = 5.0007$

$\bar{X}' = .4100$ $\bar{y} = 5.1410$

SnwXy	Snwy²	SnwX'²	SnwX'y	Snwy²
6734.2650	13329.421	84.36860	1058.0579	13279.439
6538.4472	12895.940	82.11972	1029.5862	12908.566
195.8178	433.481	2.24888	28.4717	370.873
	425.249			360.463
	χ^2_{11} = 8.232			χ^2_{11} = 10.410

$b = 12.6604$

$Y = 5.1410 + 12.6604(X - .4100)$

$Y = -.050 + 12.660X$

$X' = \dfrac{Y + .050}{12.660}$

50% pt : $X' = .40$, $X = 2.51$ $\sigma m = .004$

99% pt : $X' = .58$, $X = 3.80$ $\sigma 99 = .010$

1% pt : $X' = .21$, $X = 1.62$ $\sigma 1 = .011$

Probit Analysis of Mk 101 D
2nd Cycle

1 Height X	2 n	3 p	4 Y	5 nw	6 y	7 nwX	8 nwy	9 Log Height X'
4.00	100	100	8.2	.8	8.49	3.200	6.792	60
3.75	100	99	7.6	4.0	7.21	15.000	28.840	.57
3.50	100	99	7.1	11.0	7.28	38.500	80.080	.54
3.25	100	91	6.6	23.8	6.28	77.350	149.464	.51
3.15	56	93	6.3	18.8	6.46	59.220	121.448	.50
3.00	100	83	6.0	43.9	5.95	131.700	261.205	.48
2.80	44	73	5.6	24.6	5.61	58.880	138.006	.45
2.75	100	70	5.5	58.1	5.52	159.775	320.712	.44
2.50	312	47	4.9	197.8	4.92	494.500	973.176	.40
2.25	100	34	4.4	55.8	4.60	125.550	256.660	.35
2.00	127	11	3.8	47.0	3.77	94.000	177.190	.30
1.75	100	3	3.3	20.8	3.14	36.400	65.312	.24
1.50	100	0	2.8	9.2	2.41	13.800	22.172	.18
				515.6		1317.875	2601.077	

$\bar{X} = 2.55560$ $\bar{y} = 5.0448$

$SnwX^2$	$SnwXy$	$Snwy^2$
3460.21325	6848.3110	13565.759
3368.49207	6648.3599	13121.803
91.72118	199.9511	443.956
		435.891
		$\chi^2_{11} = 8.065$

b = 2.1800 σ = .46

Y = 5.0448+2.1800(X-2.5560)

Y = -.527+2.180X

$X = \frac{Y+.527}{2.180}$

50% pt: 2.54 in.
99% pt: 3.60 in.
1% pt; 1.47 in.

5
9

.t Analysis of Mk 101 Data
2nd Cycle

Data Sheet 2

8	9	10	11	12	13	14	15	16
nwy	Log Height X'	n	p	Y	nw	y	nwX'	nwy
6.792	60	100	100	7.5	5.0	7.85	3.000	39.250
28.840	.57	100	99	7.2	9.2	7.31	5.244	67.252
80.080	.54	100	99	6.8	18.0	7.13	9.720	128.340
49.464	.51	100	91	6.4	30.2	6.34	15.402	191.468
21.448	.50	56	93	6.3	18.8	6.46	9.400	121.448
61.205	.48	100	83	6.0	43.9	5.95	21.072	261.205
38.006	.45	44	73	5.6	24.6	5.61	11.070	138.006
20.712	.44	100	70	5.5	58.1	5.52	25.564	320.712
73.176	.40	312	47	5.0	198.7	4.92	79.480	977.604
56.660	.35	100	34	4.4	55.8	4.60	19.530	256.680
77.190	.30	127	11	3.7	42.7	3.78	12.810	161.406
65.312	.24	100	3	3.0	13.1	3.13	3.144	41.003
22.172	.18	100	0	2.2	2.5	1.88	.450	4.700
01.077					520.6		215.886	2709.074

$\bar{X} = .4147$ $\bar{y} = 5.2038$

nwy^2	$Snw X'^2$	$Snw X'y$	$Snwy^2$
65.759	92.24318	1158.1424	14549.793
21.803	89.52510	1123.4175	14097.353
43.956	2.71808	34.7249	452.440
35.891			443.629
8.065			$\chi^2_{11} = 8.811$

b = 12.7755

Y = 5.2038+12.776(X-.4147)

Y = -.094+12.776 X

$X' = \frac{Y+.094}{12.776}$

$\sigma = .078$

50% pt: X'_0 = [illegible] or 2.52 in.

99% pt: X'_0 = .58 or 3.80 in.

1% pt: X' = .22 or 1.66 in.

Probit Analysis - Mk 101 - 3rd Cycle

1 Height X	2 n	3 p	4 Y	5 nw	6 y	7 nwX	8 nwy	9 Height Y	10 Log Height X'
4.00	100	100	8.2	.8	8.49	3.200	6.792	8.2	.60
3.75	100	99	7.6	4.0	7.21	15.000	28.840	7.6	.57
3.50	100	99	7.1	11.0	7.28	38.500	80.080	7.1	.54
3.25	100	91	6.6	23.8	6.28	77.350	149.464	6.6	.51
3.15	56	93	6.3	18.8	6.46	59.220	121.448	6.3	.50
3.00	100	83	6.0	43.9	5.95	131.700	261.205	6.0	.48
2.80	44	73	5.6	24.6	5.61	68.880	138.006	5.6	.45
2.75	100	70	5.5	58.1	5.52	159.775	320.712	5.5	.44
2.50	312	47	4.9	197.8	4.92	494.500	973.176	4.9	.40
2.25	100	34	4.4	55.8	4.60	125.550	256.680	4.4	.35
2.00	127	11	3.8	47.0	3.77	94.000	177.190	3.8	.30
1.75	100	3	3.3	20.8	3.14	36.400	65.312	3.3	.24
1.50	100	0	2.7	7.6	2.32	11.400	17.632	2.7	.18
				514.0		1315.475	2596.537		

$\bar{X} = 2.5593$ $\bar{y} = 5.0516$

$SnwX^2$	$SnwXy$	$Snwy^2$
3456.61325	6841.5010	13553.231
3366.68185	6645.2909	13116.740
89.93140	196.2101	436.491
		428.086
		$\chi^2_{11} = 8.405$

$b = 2.1818$ $\sigma = .46$

$Y = 5.0516 + 2.1818(X - 2.5593)$

$Y = -.532 + 2.182X$

$X = \frac{Y + .532}{2.182}$

50% pt: 2.54 in. $\sigma m = .020$ (2.49-2.59

99% pt: 3.60 in. $\sigma 99 = .054$ 99% C.I. (3.46-3.74

1% pt: 1.47 in. $\sigma 1 = .057$ (1.32-1.62

Analysis - Mk 101 - 3rd Cycle

Data Sheet 3

8	9	10	11	12	13	14	15	16	17
	Height	Log Height							
nwy	Y	X'	n	p	Y	nw	y	nwX'	nwy
6.792	8.2	.60	100	100	7.6	4.0	7.94	2.400	31.760
28.840	7.6	.57	100	99	7.2	9.2	7.31	5.244	67.252
80.080	7.1	.54	100	99	6.8	18.0	7.13	9.720	128.340
49.464	6.6	.51	100	91	6.4	30.2	6.34	15.402	191.468
21.448	6.3	.50	56	93	6.3	18.8	6.46	9.400	121.448
61.205	6.0	.48	100	83	6.0	43.9	5.95	21.072	261.205
38.006	5.6	.45	44	73	5.7	23.4	5.61	10.530	131.274
20.712	5.5	.44	100	70	5.5	58.1	5.52	25.564	320.712
73.176	4.9	.40	312	47	5.0	198.7	4.92	79.480	977.604
56.680	4.4	.35	100	34	4.4	55.8	4.60	19.530	256.680
77.190	3.8	.30	127	11	3.7	42.7	3.78	12.810	161.406
65.312	3.3	.24	100	3	3.0	13.1	3.13	3.144	41.003
17.632	2.7	.18	100	0	2.2	2.5	1.88	0.450	4.700
96.537						518.4		214.746	2694.852

$\bar{X}$ = .4142 $\bar{y}$ = 5.1984

$Snwy^2$	$SnwX'^2$	SnwX'y	$Snwy^2$
53.231	91.64018	1150.6190	14456.089
16.740	88.95803	1116.3362	14008.926
36.491	2.68215	34.2828	447.163
28.086			438.376
8.405			χ^2_{11} = 8.787

b = 12.7818

Y = 5.1984+12.7818(X' -.4142)

Y = -.096+12.782X'

X = $\frac{Y+.096}{12.782}$

50% pd:X = .399 or 2.51 in. σm = .003

99% pd:X = .581 or 3.81 in. σ99 = .009

1% pd:X = .217 or 1.65 in. σ1 = .010

(2.46-2.55

90% C.I. (3.62-4.02

(1.55-1.75

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3

Data Sheet 4

χ^2 Tests of Mk 101 Data

(200-trial Bruceton Tests)

$m = 2.55$
$\sigma = .32$

1/4 inch

1 i	2 ni	3 hi	4 $\frac{hi-m}{\sigma}$	5 pi	6 qi	7 qi/pi	8 wi	9 ni'	10 $\frac{(ni-ni')^2}{ni'}$
0	1	2.00	-1.72				1.000	6.9	.64
1	29	2.25	-.94	.174	.826	4.747	4.747	32.8	.44
2	44	2.50	-.16	.436	.564	1.294	6.143	42.4	.06
3	16	2.75	.62	.732	.268	.366	2.248	15.5	.02
4	1	3.00	1.41	.921	.079	.086	.193	1.3	.24
	99						14.331		χ^2_2 = 1.40 P=.50

$m = .397$
$\sigma = .087$

.05 log

1 i	2 ni	3 hi	4 $\frac{hi-m}{\sigma}$	5 pi	6 qi	7 qi/pi	8 wi	9 ni'	10 $\frac{(ni-ni')^2}{ni'}$
0	2	.255					1.000	2.3	.04
1	20	.301	-1.10	.136	.864	6.353	6.353	14.6	2.00
2	33	.352	-.52	.302	.698	2.311	14.682	33.8	.02
3	31	.398	.01	.504	.496	.984	14.447	33.2	.15
4	12	.447	.57	.716	.284	.397	5.735	13.2	.11
5	1	.498	1.16	.877	.123	.140	.803	1.8	.54
	99						43.020		χ^2_3 = 2.86 P=.42

$m = 2.60$
$\sigma = .42$

1/2 inch

1 i	2 ni	3 hi	4 $\frac{hi-m}{\sigma}$	5 pi	6 qi	7 qi/pi	8 wi	9 ni'	10 $\frac{(ni-ni')^2}{ni'}$
0	2	1.50					1.000	2.9	.28
1	37	2.00	-1.43	.076	.924	12.158	12.158	35.0	.11
2	49	2.50	-.24	.405	.595	1.469	17.860	51.4	.11
3	12	3.00	.95	.829	.171	.206	3.679	10.6	.18
	100						34.697		χ^2_1 = .68 P=.43

$m = .390$
$\sigma = .084$

.10 log

1 i	2 ni	3 hi	4 $\frac{hi-m}{\sigma}$	5 pi	6 qi	7 qi/pi	8 wi	9 ni'	10 $\frac{(ni-ni')^2}{ni'}$
0	10	.204					1.000	8.0	.50
1	47	.301	-1.06	.145	.855	5.897	5.897	47.3	.0C
2	40	.398	.10	.540	.460	.852	5.024	40.3	.00
3	3	.498	1.29	.901	.099	.110	.553	4.4	.45
	100						12.474		χ^2_1 = .95 P=.54

Probit Analysis of Mk [illegible]
1st Cycle

1 Height X	2 n No. of Trials	3 r No. of Explo.	4 p % Explo.	5 Empirical Probit	6 Y Expected Probit	7 nw	8 y Working Probit	9 nwX
8.50	40	40	100	∞	7.4	2.5	7.77	21.250
8.00	16	14	88	6.18	7.2	1.5	4.21	12.000
7.50	50	49	98	7.05	6.9	7.7	7.03	57.750
6.50	50	43	86	6.08	6.3	16.8	6.05	109.200
6.00	50	43	86	6.08	6.0	22.0	6.08	132.000
5.50	63	50	79	5.81	5.7	33.5	5.80	184.250
5.00	96	46	48	4.95	5.4	57.7	4.92	288.500
4.55	41	25	61	5.28	5.1	26.0	5.28	118.300
4.50	50	21	42	4.80	5.1	31.7	4.80	142.650
4.15	38	14	37	4.67	4.8	23.8	4.67	98.770
4.00	50	22	44	4.85	4.8	31.4	4.85	125.600
3.80	19	6	32	4.53	4.6	11.4	4.53	43.320
3.50	50	17	34	4.59	4.5	29.0	4.59	101.500
2.50	50	9	18	4.08	3.9	20.2	4.10	50.500
2.00	50	1	2	2.95	3.6	15.1	3.19	30.200
1.50	50	1	2	2.95	3.3	10.4	3.04	15.600
						340.7		1531.390

$\bar{x} = 4.4948$ $\bar{y} = 4.9712$

$SnwX^2$	SnwXy
7489.82650	7965.6015
6883.34409	7612.8467
606.48241	352.7548

$\chi^2_{14} =$

b = .5816
Y = 4.9712+.5816 [illegible]
= 2.36+.58X

$X = \frac{Y-2.36}{.58}$

50% pt: X = 4.55
99% pt: X = 8.57
1% pt: X = .53

nalysis of Mk 120 Data

1st Cycle

Data Sheet 5

9	10	11	12	13	14	15	16
mwX	mwy	X' Log Height	Y	mw	y	mwX'	mwy
21.250	19.425	.93	6.7	8.3	7.17	7.719	59.511
12.000	6.315	.90	6.6	3.8	6.01	3.420	22.838
57.750	54.131	.88	6.4	15.1	6.81	13.288	102.831
109.200	101.640	.81	6.0	22.0	6.08	17.820	133.760
132.000	133.760	.78	5.8	25.2	6.05	19.656	152.460
184.250	194.300	.74	5.6	35.2	5.79	26.048	203.808
288.500	283.884	.70	5.4	57.7	4.92	40.390	283.884
118.300	137.280	.66	5.1	26.0	5.28	17.160	137.280
142.650	152.[illegible]60	.65	5.1	31.7	4.80	20.605	152.160
98.770	111.146	.62	4.9	24.1	4.67	14.942	112.547
125.600	152.290	.60	4.8	31.4	4.85	18.[illegible]	152.290
43.320	51.642	.58	4.7	11.7	4.54	6.786	53.118
101.500	133.110	.54	4.4	27.9	4.60	15.066	128.340
50.500	82.820	.40	3.6	15.1	4.26	6.040	64.326
30.200	48.169	.30	3.0	6.6	2.95	1.980	19.470
15.600	31.616	.18	2.3	1.6	3.89	.[illegible]	6.224
1531.390	1693.636			343.4		230.048	1784.847

$\bar{y} = 4.9712$

$\bar{X}' = .6712$ $\qquad \bar{y}' = 5.1976$

$Smwy^2$

8657.007

8419.663

237.340

$\chi^2_{14} = \frac{205.177}{32.163}$

$SmwX'y$

1226.6575

1195.6916

30.9659

[illegible] (.6712)

$Smwy^2$

9483.234

9276.875

206.359

$\chi^2_{14} = \frac{172.069}{34.290}$

[illegible] = .639 or 4.32 in.

99% [illegible] = 1.054 or 11.32 in.

1% [illegible] = .225 or 1.64 in.

1 Height X	2 r	3 p	4 Y	5 nw	6 y	7 nwX	8 nwy	9 Log Height X'
8.50	[illegible]	100	7.3	3.0	7.68	25.500	23.040	.93
8.00	[illegible]	88	7.0	2.1	5.20	16.800	10.920	.90
7.50	[illegible]	98	6.7	10.4	6.96	78.000	72.384	.88
6.50	[illegible]	86	6.1	20.2	6.08	131.300	122.816	.81
6.00	[illegible]	85	5.8	25.2	6.05	151.200	152.460	.78
5.50	[illegible]	79	5.6	35.2	5.79	193.600	203.808	.74
5.00	[illegible]	48	5.3	59.1	4.94	295.500	291.954	.70
4.55	[illegible]	61	5.0	26.1	5.28	118.755	137.808	.66
4.50	[illegible]	42	5.0	31.8	4.80	143.100	152.640	.65
4.15	[illegible]	37	4.8	23.8	4.67	98.770	111.146	.62
4.00	[illegible]	44	4.7	30.8	4.85	123.200	149.380	.60
3.80	[illegible]	32	4.6	11.4	4.53	43.320	51.642	.58
3.50	[illegible]	34	4.4	27.9	4.60	97.650	128.340	.54
2.50	[illegible]	18	3.8	18.5	4.13	46.250	76.405	.40
2.00	[illegible]	2	3.5	13.4	3.14	26.800	42.076	.30
1.50	[illegible]	2	3.2	9.0	3.00	13.500	27.000	.18
				347.9		1603.245	1753.819	

$\bar{x}$ = [illegible].6084 $\bar{y}$ = 5.0412

$SnwX^2$	SnwXy	$Snwy^2$
8021.9[illegible]	[illegible].6394	9085.746
7388.3[illegible]	[illegible].8120	8841.279
633.6[illegible]	[illegible]	244.467
		217.720

χ^2_{14} = 26.747

b .5862
Y 5.0412+.5862(X-[illegible]
 2.34+.59 X
X Y-2.34 / .59 Uncorrected

50 4.53 in. σm = .09
[illegible] .56 in. σ1 = .29

Heterogeneity Factor T.0[illegible]

V(m) = .0084 x 1.69 = .0142) σm = .12 x [illegible]
V(1) = .0835 x 1.69 = .1416 σ1 = .38 x [illegible]

obit Analysis of Mk 120 Data
2nd Cycle

	9	10	11	12	13	14	15	16
	Log Height X'	n	p	Y	nw	y	nwX'²	nwy
0	.93	40	100	6.6	9.5	7.09	8.835	67.355
0	.90	16	88	6.5	4.3	6.09	3.870	26.187
4	.88	50	98	6.4	15.1	6.81	13.288	102.831
6	.81	50	86	6.0	22.0	6.08	17.820	133.760
0	.78	50	86	5.8	25.2	6.05	19.656	152.460
8	.74	63	79	5.6	35.2	5.79	26.048	203.808
4	.70	96	48	5.4	57.7	4.92	40.390	283.884
8	.66	41	61	5.1	26.0	5.28	17.160	137.280
0	.65	50	42	5.1	31.7	4.80	20.605	152.160
6	.62	38	37	4.9	24.1	4.67	14.942	112.547
0	.60	50	44	4.8	31.4	4.85	18.840	152.290
2	.58	19	32	4.7	11.7	4.54	6.786	53.118
0	.54	50	34	4.5	29.0	4.59	15.660	133.110
5	.40	50	18	3.7	16.8	4.18	6.720	70.224
6	.30	50	2	3.1	7.7	2.97	2.310	22.869
0	.18	50	2	2.5	2.5	3.29	.450	8.225
9					349.9		233.380	1812.108

$\bar{X}' = .6670$ $\bar{y} = 5.1789$

6	$SnwX'^2$	$SnwX'y$	$Snwy^2$
9	161.[illegible]	1213.2814	9609.762
7	155.[illegible]	1208.6589	9384.783
0	[illegible]	4.6225	224.979
7			193.770
			$\chi^2_{14} = 31.209$

b = [illegible]
Y = [illegible] [illegible]
= [illegible]
X' = [illegible]
[illegible]

$T_{.01}$ for 9 d. f.

[illegible]) σ_m = .12 x 3.25 = .39 } for 50%: .65 or 4.27 in.
[illegible] = .38 x 3.25 = 1.24) [illegible] 1%: .22 or 1.66 in.

χ^2 Tests of Mk 120 Data

1 Height X	2 Y	3 P	4 n	5 r	6 nP	7 r-nP	8 $\frac{(r-nP)^2}{nP}$	9 Log Height X'
8.50	7.36	99.1	40	40	39.6			.93
8.00	7.06	98.0	16	14	15.7			.90
7.50	6.76	96.1	50	49	48.0	1.3		.88
6.50	6.18	88.1	50	43	44.0		.21	.81
6.00	5.88	81.1	50	43	40.6	2.4	.75	.78
5.50	5.58	71.9	63	50	45.3	4.7	1.74	.74
5.00	5.29	61.4	96	46	58.9	12.9	7.31	.70
4.55	5.02	50.8	41	25	20.8	4.2	1.72	.66
4.50	5.00	50.0	50	21	25.0	4.0	1.28	.65
4.15	4.79	41.7	38	14	15.8	1.8	.35	.62
4.00	4.70	38.2	50	22	19.1	2.9	.71	.60
3.80	4.58	33.7	19	6	6.4	.4	.04	.58
3.50	4.40	27.4	50	17	13.7	3.3	1.09	.54
2.50	3.82	11.9	50	9	6.0		.01	.40
2.00	3.52	6.9	50	1	3.4	.3		.30
1.50	3.22	3.8	50	1	1.9			.18

$\chi^2_9 = 15.21$

of Mk 120 Data

Data Sheet 7

8	9	10	11	12	13	14	15
$(r-nP)^2$ / [illegible]	Log Height X'	Y	P	n			
	.93	6.66	95.2	40			
	.90	6.49	93.2	16	14	14.9)	
	.88	6.38	91.6	50	49	45.8)	5.2
.21	.81	5.99	83.9	50	43	42.0)	
.75	.78	5.82	79.4	50	43	39.7	3.3
1.74	.74	5.59	72.2	63	50	45.5	4.5
7.31	.70	5.37	64.4	96	46	61.8	15.8
1.72	.66	5.15	56.0	41	25	23.0	2.0
1.28	.65	5.09	53.6	50	21	26.8	5.8
.35	.62	4.92	46.8	38	14	17.8	3.8
.71	.60	4.81	42.5	50	22	21.2	0.8
.04	.58	4.70	38.2	19	6	7.3	1.3
1.09	.54	4.47	29.8	50	17	14.9	2.1
.01	.40	3.69	9.5	50	9	4.8)	
	.30	3.13	3.1	50	1	1.6)	4.3
	.18	2.46	0.6	50	1	.3)	

15.21

The values of r-nP are on the whole much greater than for height, so the calculation was not completed.

1 Height X	2 n	3 r	4 p
4.50	100	100	100
4.25	100	97	97
4.00	100	97	97
3.75	100	94	94
3.50	100	94	94
3.25	100	97	97
3.00	100	93	93
2.75	100	89	89
2.50	100	78	78
2.25	100	62	62
2.00	100	50	50
1.75	100	37	37
1.50	100	19	19
1.25	100	11	11
1.00	100	5	5
.75	100	0	0

$m = 4.86$
$\sigma = 1.51$

1/2 inch

χ^2

1 i	2 ni	3 hi'	4 $\frac{hi-m}{\sigma}$	5 pi
0	1	3.00		
1	8	3.50	-.90	.184
2	21	4.00	-.57	.284
3	29	4.50	-.24	.405
4	27	5.00	.09	.536
5	8	5.50	.42	.663
6	4	6.00	.75	.773
7	2	6.50	1.09	.862
	100			

$m = .651$
$\sigma = .220$

0	1	.415		
1	2	.455	-.89	.187
2	5	.498	-.70	.242
3	5	.538	-.51	.305
4	13	.580	-.32	.374
5	24	.618	-.15	.440
6	16	.658	.03	.512
7	23	.699	.22	.587
8	5	.740	.40	.655
9	2	.782	.60	.726
10	1	.823	.78	.782
	97			

χ^2 Tests of Mk 120 Data
(200-trial Bruceton Tests)

Data Sheet 8

4	5	6	7	8	9	10	
$\frac{hi-m}{\sigma}$	pi	qi	qi/pi	wi	ni'	$\frac{(na-ni')^2}{ni}$	
				1.000	1.8	.54	
-.90	.184	.816	4.435	4.435	7.8	.01	
-.57	.284	.716	2.521	11.181	19.6	.10	
-.24	.405	.595	1.469	16.425	28.8	.00	
.09	.536	.464	.866	14.224	25.0	.16	
.42	.663	.337	.508	7.226	12.7	1.74	
.75	.773	.227	.294	2.124	3.7	.02	
1.09	.862	.138	.160	.340	.6	2.39	P = .43
				56.955	χ^2_5 =	4.96	

				1.000	.3	1.27	
-.89	.187	.813	4.348	4.348	1.4	.68	
-.70	.242	.758	3.132	13.618	4.3	.11	
-.51	.305	.695	2.279	31.035	9.7	2.28	
-.32	.374	.626	1.674	51.953	16.3	.67	
-.15	.440	.560	1.273	66.136	20.7	.53	
.03	.512	.488	.953	63.028	19.7	.69	
.22	.587	.413	.704	44.372	13.9	5.96	
.40	.655	.345	.527	23.384	7.3	.72	
.60	.726	.274	.377	8.816	2.8	.23	
.78	.782	.218	.279	2.460	.8	.36	P = .10
				310.150	χ^2_8 =	13.50	

Probit Analysis of Mk 10

1 Height X	2 n	3 r	4 p	5 Empirical Probit	6 Y	7 nw	8 y	9 nwX	10 nwy	11 1+Log
4.50	100	100	100		7.6	4.0	7.94	18.000	31.760	1
4.25	100	97	97	6.88	7.3	7.6	6.62	32.300	50.312	1
4.00	100	97	97	6.88	7.1	11.0	6.82	44.000	75.020	1
3.75	100	94	94	6.55	6.8	18.0	6.50	67.500	117.000	1
3.50	100	94	94	6.55	6.5	26.9	6.55	94.150	176.195	[illegible]
3.25	100	97	97	6.88	6.3	33.6	6.69	109.200	224.784	[illegible]
3.00	100	93	93	6.48	6.0	43.9	6.37	131.700	279.643	[illegible]
2.75	100	89	89	6.23	5.8	50.3	6.15	138.325	309.345	[illegible]
2.50	100	78	78	5.77	5.5	58.1	5.75	145.250	334.075	1
2.25	100	62	62	5.31	5.2	62.7	5.30	141.075	332.310	1
2.00	100	50	50	5.00	4.9	63.4	5.00	126.800	317.000	1
1.75	100	37	37	4.67	4.7	61.6	4.67	107.800	287.672	1
1.50	100	19	19	4.12	4.4	55.8	4.15	83.700	231.570	[illegible]
1.25	100	11	11	3.77	4.2	50.3	3.85	62.875	193.655	[illegible]
1.00	100	5	5	3.36	3.9	40.5	3.51	40.500	142.155	[illegible]
.75	100	0	0	—	—	—	—	—	—	
						587.7		1343.175	3102.496	

$\bar{X} = 2.2855$ $\bar{y} = 5.2790$

$SnwX^2$	$SnwXy$	$Snwy^2$
[illegible].756	7584.324	17015.210
[illegible].756	7090.684	16378.223
[illegible]	493.640	636.987
		601.740
		$\chi^2_{13} = 35.247$

b = [illegible]

Y = 5.2790+1.2190(X-2.2855)

= 2.49+1.22X

$X = \frac{Y-2.49}{1.22}$

50% pt: 2.06 in.

99% pt: 3.97 in.

1% pt: .15 in.

Probit Analysis of Mk 102 Data

9	10	11	12	13	14	15	16
		1+Log Height					
nwX	nwy	X'	Y	nw	y	nwX'	nwy'
18.000	31.760	1.65	7.3	7.6	7.68	12.540	58.368
32.300	50.312	1.63	7.2	9.2	6.75	14.996	62.100
44.000	75.020	1.60	7.0	13.1	6.87	20.960	89.997
67.500	117.000	1.57	6.8	18.0	6.50	28.260	117.000
94.150	176.195	1.54	6.6	23.8	6.55	36.652	155.890
109.200	224.784	1.51	6.4	30.2	6.74	45.602	203.548
131.700	279.643	1.48	6.2	37.0	6.43	54.760	237.910
138.325	309.345	1.44	6.0	43.9	6.20	63.216	272.180
145.230	334.075	1.40	5.7	53.2	5.77	74.480	306.964
141.075	332.310	1.35	5.4	60.1	5.30	81.135	318.530
126.800	317.000	1.30	5.1	63.4	5.00	82.420	317.000
107.800	287.672	1.24	4.7	61.6	4.67	76.384	287.672
83.700	231.570	1.18	4.3	53.2	4.13	62.776	219.716
62.875	193.655	1.10	3.8	37.0	3.77	40.700	139.490
40.500	142.155	1.00	3.2	18.0	3.38	18.000	60.840
1343.175	3102.496			529.3		712.881	2847.205

5.2790

$\bar{X}' = 1.3468$ $\quad \bar{y} = 5.3792$

$Snwy^2$
17015.210
16378.223
636.987
601.740
$\chi^2_{13} = 35.247$

$SnwX'^2$
972.5[illegible]
960.[illegible]
12.[illegible]

$SnwX'y$
3918.812
3834.722
84.090

$Snwy^2$
15883.254
15315.655
567.599
552.648
$\chi^2_{13} = 14.951$

b = 6.572[illegible]
Y = 5.3792 + 6.5721(X' - 1.3468)
Y = -3.47 + 6.57X'
$X' = \frac{Y + 3.47}{6.57}$

50% pt: 1.99 in.
99% pt: 4.41 in.
1% pt: .86 in.

$m = 4.86$
$\sigma = 1.51$

1/2 inch

χ^2 Tests of Mk 120 Data
(200-trial Bruceton Tests)

Data Sheet 8

1	2	3	4	5	6	7	8	9	10	
i	ni	hi'	$\frac{hi-m}{\sigma}$	pi	qi	qi/pi	wi	ni'	$\frac{(ni-ni')^2}{ni}$	
0	1	3.00					1.000	1.8	.54	
1	8	3.50	-.90	.184	.816	4.435	4.435	7.8	.01	
2	21	4.00	-.57	.284	.716	2.521	11.181	19.6	.10	
3	29	4.50	-.24	.405	.595	1.469	16.425	28.8	.00	
4	27	5.00	.09	.536	.464	.866	14.224	25.0	.16	
5	8	5.50	.42	.663	.337	.508	7.226	12.7	1.74	
6	4	6.00	.75	.773	.227	.294	2.124	3.7	.02	
7	2	6.50	1.09	.862	.138	.160	.340	.6	2.39	P = .43
	100						56.955	χ^2_5	= 4.96	

$m = .651$
$\sigma = .220$

1	2	3	4	5	6	7	8	9	10	
0	1	.415					1.000	.3	1.27	
1	2	.455	-.89	.187	.813	4.348	4.348	1.4	.68	
2	5	.498	-.70	.242	.758	3.132	13.618	4.3	.11	
3	5	.538	-.51	.305	.695	2.279	31.035	9.7	2.28	
4	13	.580	-.32	.374	.626	1.674	51.953	16.3	.67	
5	24	.618	-.15	.440	.560	1.273	66.136	20.7	.53	
6	16	.658	.03	.512	.488	.953	63.028	19.7	.69	
7	23	.699	.22	.587	.413	.704	44.372	13.9	5.96	
8	5	.740	.40	.655	.345	.527	23.384	7.3	.72	
9	2	.782	.60	.726	.274	.377	8.816	2.8	.23	
10	1	.823	.78	.782	.218	.279	2.460	.8	.36	P = .10
	97						310.150	χ^2_8	= 13.50	

Probit Analysis of Mk 102 Data

2nd Cycle

1 Height X	2 Y	3 nw	4 y	5 nwX	6 nwy
4.50	8.0	1.5	8.30	6.750	12.450
4.25	7.7	3.1	5.15	13.175	15.965
4.00	7.4	6.2	6.43	24.800	39.866
3.75	7.0	13.1	6.31	49.125	82.661
3.50	6.8	18.0	6.50	63.000	117.000
3.25	6.4	30.2	6.74	98.150	203.548
3.00	6.2	37.0	6.43	111.000	237.910
2.75	6.3	33.6	6.22	92.400	208.992
2.50	5.6	55.8	5.76	139.500	321.408
2.25	5.2	62.7	5.30	141.075	332.310
2.00	4.9	63.4	5.00	126.800	317.000
1.75	4.6	60.1	4.67	105.175	280.667
1.50	4.3	53.2	4.13	79.800	219.716
1.25	4.0	43.9	3.80	54.875	166.820
1.00	3.7	33.6	3.43	33.600	115.248
		515.4		1139.225	2671.561

$\bar{X} = 2.2104$ $\qquad \bar{y} = 5.1835$

$S_{nwX^2} = \dfrac{2822.094 - 2518.109}{303.985}$ $\qquad S_{nwXy} = \dfrac{6282.540 - 5905.140}{377.400}$ $\qquad S_{nwy^2} = \dfrac{14370.501 - 13847.959}{522.542}$

$$\chi^2_{13} = \frac{468.545}{53.997}$$

$b = 1.2415$

$Y = 5.1835 + 1.2415\ (X - 2.2104)$

$\quad = 2.44 + 1.24X$

Data Sheet 11

χ^2 Tests of Mk 102 Data

1	2	3	4	5	6	7	8
Height X	Y	P	n	r	nP	r-nP	$\frac{(r-nP)^2}{nP(1-P)}$
4.50	7.98	99.8	100	100	99.8)		
4.25	7.67	99.6	100	97	99.6)	4.5	13.56
4.00	7.37	99.1	100	97	99.1)		
3.75	7.07	98.0	100	94	98.0	4.0	8.16
3.50	6.76	96.1	100	94	96.1)	2.3	.50
3.25	6.45	92.6	100	97	92.6)		
3.00	6.15	87.5	100	93	87.5	5.5	2.77
2.75	5.85	80.2	100	89	80.2	8.8	4.88
2.50	5.54	70.5	100	78	70.5	7.5	2.70
2.25	5.23	59.1	100	62	59.1	2.9	.35
2.00	4.93	47.2	100	50	47.2	2.8	.31
1.75	4.63	35.6	100	37	35.6	1.4	.08
1.50	4.32	24.8	100	19	24.8	5.8	1.80
1.25	4.01	16.1	100	11	16.1)		
1.00	3.71	9.9	100	5	9.9)	10.0	4.42
.75	3.41	5.6	100	0	5.6		

$\chi^2_9 = 39.53$